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RESULTS OF RESEARCH ON SURFACE WAVES OF THE WESTERN NORTH ATLANTIC

I

INVESTIGATION OF BOTTOM PRESSURE FLUCTUATIONS AND SURFACE WAVES

II

RESULTS OF SEA SURFACE ROUGHNESS DETERMINATIONS IN THE VICINITY OF WOODS HOLE, MASSACHUSETTS AND BERMUDA

BY

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PART I

INVESTIGATION OF BOTTOM PRESSURE FLUCTUATIONS AND SURFACE WAVES

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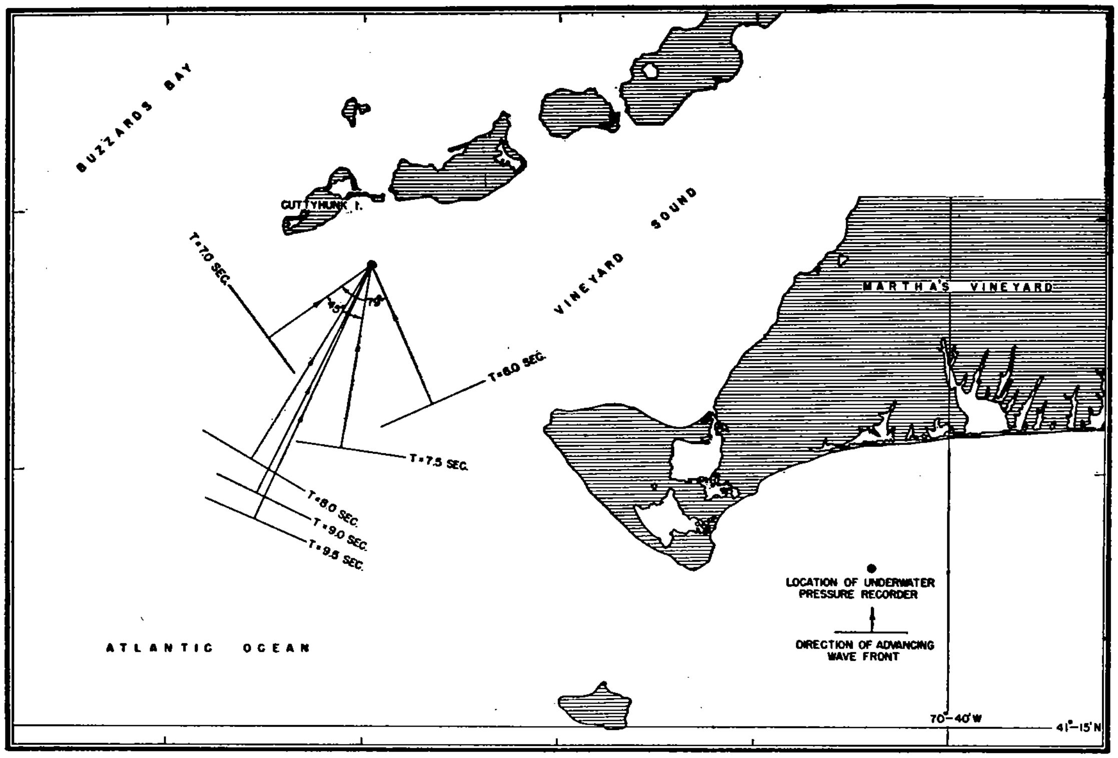

FIG. 1. Location of Underwater Pressure Recorder off Cuttyhunk Island (June 1946), and directions of advancing wave fronts, Experiment II (see text).

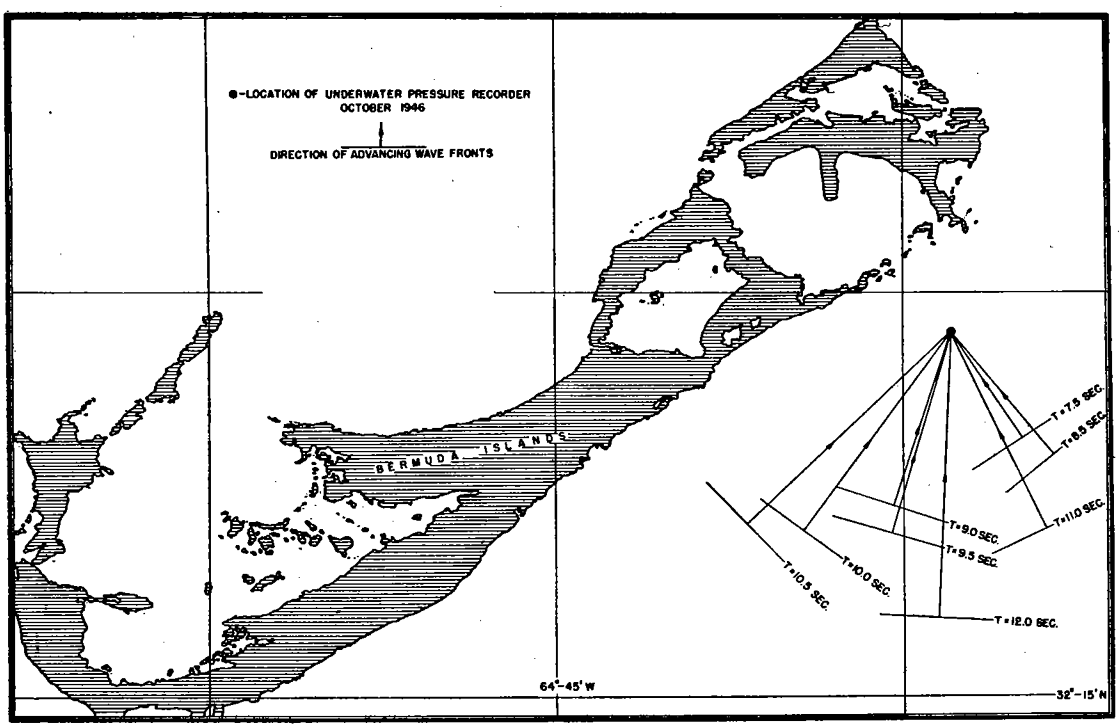

FIG. 2. Location of Underwater Pressure Recorder off Bermuda (October 1946), and directions of advancing wave fronts, Experiment V (see text).

PART I

INVESTIGATION OF SIMULTANEOUS PRESSURE FLUCTUATIONS AT THE BOTTOM AND SURFACE OF THE OCEAN

I. INTRODUCTION

It is to be expected that in the future, measurements of pressure fluctuations beneath the ocean surface will provide basic data for solution of many practical problems on the state of the sea. As a result of the wartime impetus, several types of underwater pressure recorders (of similar instrumentation principles) were developed both in this country and abroad. Essentially, the instrument consists of an underwater unit[1] which electrically transmits pressure impulses near the sea bottom to a clockwork recorder installed on the shore. The underwater pressure unit is adjusted for pressure fluctuations resulting from surface waves within the spectrum band of periods set up by winds acting on the sea surface. The resulting records may be scaled for height and period of the pressure fluctuations over known time intervals. An accessory wave analyzer has been constructed for rapid periodogram analysis of the pressure records.

This investigation is concerned with a comparative study of observed sea surface waves and recorded sea bottom pressure fluctuations. It was undertaken for the purpose of evaluating sea surface wave heights from sea bottom pressure recordings in the vicnity of Woods Hole and Bermuda[2].

II. THE PROBLEM

Amplitudes of pressure fluctuations within the sea are not identical with those of the overlying physical surface, and records of such need be considered in light of the hydrodynamic properties of the water column. The damping of surface pressure fluctuations with depth is related to wave length and depth. The damping phenomenon is selective to the extent that longer period waves, generally obscured at the sea surface, become recognized in the underwater pressure records. However, by the same token, shorter period waves are eliminated, and if the location of the instrument is too deep, the resulting records may be unsuitable for determination of operational interference patterns at the sea surface. Hence, at the outset, it is required to investigate quantitative relationships of simultaneous pressure fluctuations at the surface and bottom and to find a means for determining sea surface patterns solely from underwater pressure records.

In view of the increasing requirements for quantitative data on height, period, steepness, and other physical characteristics of surface waves, the problem of recording and analyzing the observations has become of first order importance in physical oceanography. The present investigation is fundamental, in that it provides a method for evaluating sea surface conditions from continuous underwater pressure records. Basic data, illustrating conditions of the experiments, are given in detail for use of future related work.

[1] The underwater pressure instrument was constructed by Mr. Arthur A. Klebba under U. S. Navy Contract NObs-2083 at the Woods Hole Oceanographic Institution. This instrument is based on an original design by Dr. Maurice Ewing of this Institution.

[2] Sea surface observations and simultaneous underwater pressure records were obtained with the combined facilities of the Woods Hole Oceanographic Institution and those provided by U. S. Navy Contract NObs-2083.

III. EXPERIMENTAL METHODS

The approach to the problem has been to compare theoretical sea surface wave heights, computed from amplitudes and periods of bottom pressure fluctuations (recorded by the underwater pressure meter), with observed heights and periods of waves at the overlying sea surface. The evaluation of differences between observed and theoretical values permits computation of empirical correction factors, and their probable errors, for later use in estimating state of the sea surface from underwater pressure recordings alone.

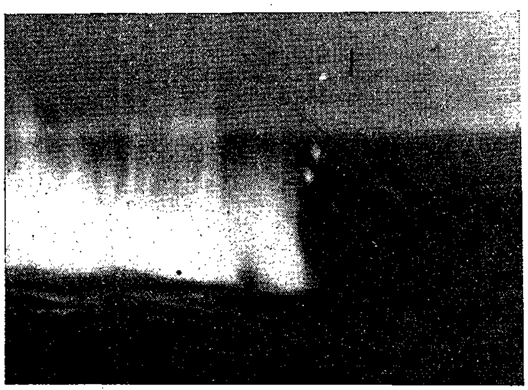

FIG. 3. Sea surface geyser from exploding 75 feet of primacord; off Cuttyhunk, June 1946.

The investigation comprised three series of experiments, two in the vicinity of Woods Hole (off Cuttyhunk Island) and one at Bermuda. The Woods Hole location was one and one-quarter miles south of Cuttyhunk Island (Figure 1), and the Bermuda location about three-fourths of a mile southeast of Castle Harbor entrance (Figure 2). At each location, an underwater pressure recording unit had been in operation for several months.

Measurements of sea surface waves over the underwater instruments were obtained by photographing changes in sea surface height against an anchored floating graduated

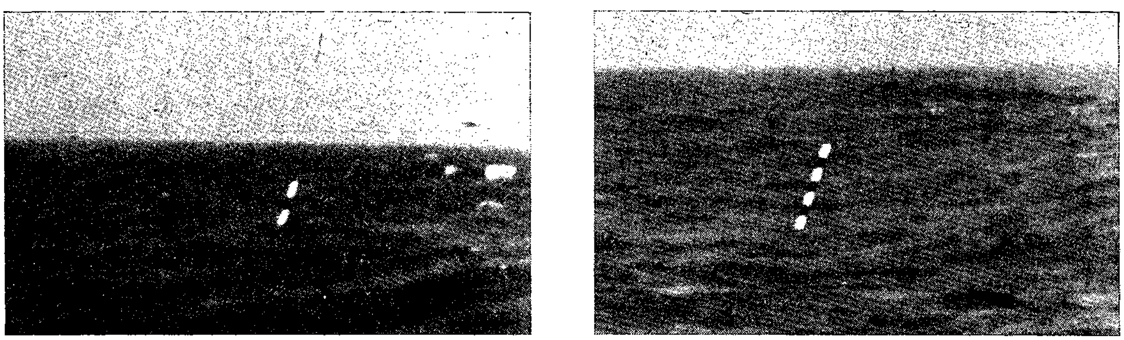

FIG. 4. Two successive relative positions of graduated pole resulting from sea surface fluctuations, off Cuttyhunk, June 27, 1946.

pole with a 16 millimeter motion picture camera (Figures 3 and 4.) The pole was anchored about fifty feet distant from the underwater instrument[3]. It was positioned

[3] This distance was computed from instrument location data, provided by Navy Contract NObs-2083, taking into account directions of the tidal currents.

with reference to a can buoy marking the underwater pressure unit. Synchronization of observations was obtained by exploding seventy-five feet of single strand primacord, shortly after the camera was set in operation. The explosion set up a geyser at the sea surface to be photographed against the background of the graduated pole (Figure 3), and a recognizable tick was simultaneously produced on the underwater recorder tape (Figure 5). The camera speed was accurately known; nearly 16 frames per second; each photographic run was for approximately 50 seconds. After processing, the individual frames were scaled and plotted against time[4]; the curves represent actual time changes in the sea surface patterns. The floating pole was graduated into six inch alternate black and white divisions. Scalings of water levels from enlarged films were made to one-tenth of a division, or one-twentieth of a foot. The probable error of the observations is one-twentieth (1/20) foot.

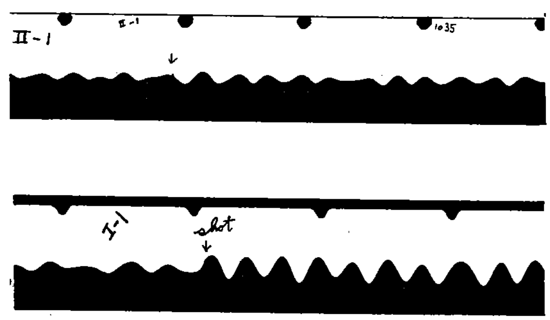

FIG. 5. Underwater recorder tapes for Experiments I-1 and II-1 showing explosion ticks.

TABLE I

EXPERIMENT	PHOTO RUN	STARTING TIME	FRAMES EXPOSED	TOTAL TIME	AVERAGE HEIGHT	AVERAGE PERIOD
I	1	10 58 21	761			
Cuttyhunk	2	11 10 32	759			
June 14	5	11 48 52	759			
	6	12 01 26	570	1h 4m	1.19 ft.	4.34 secs.
II	1	10 33 30	759			
Cuttyhunk	2	10 48 51	759			
June 27	3	10 52 07	754		(1.10 ft.)	(2.8)
	5	11 11 01	758		(2.00 ft.)	(3.7)
	6	11 21 39	760	Oh 49m	1.42 ft.	3.16 secs.
V	1	14 53 00	700			
Bermuda	2	14 57 00	740			
October 25	5	15 12 00	740	Oh 20m	1.5 ft.	3.0 secs.

Field experimental data and average crest height and period of surface waves during observational time. Sub averages bracketed in Experiment II for 10h33′30″ to 10h52′07″ and 11h11′01″ to 11h21′39″.

Data for the Cuttyhunk experiments (I and II) are shown in Figures 6 to 14, that for Bermuda (Experiment V) in Figure 15. The reproduced underwater records for Experiments I and II are also shown in Figures 6 to 14. Table 1 summarizes the field data of the experiments.

[4] Experiments in which the floating graduated pole leaned more than fifteen degrees from the vertical were rejected.

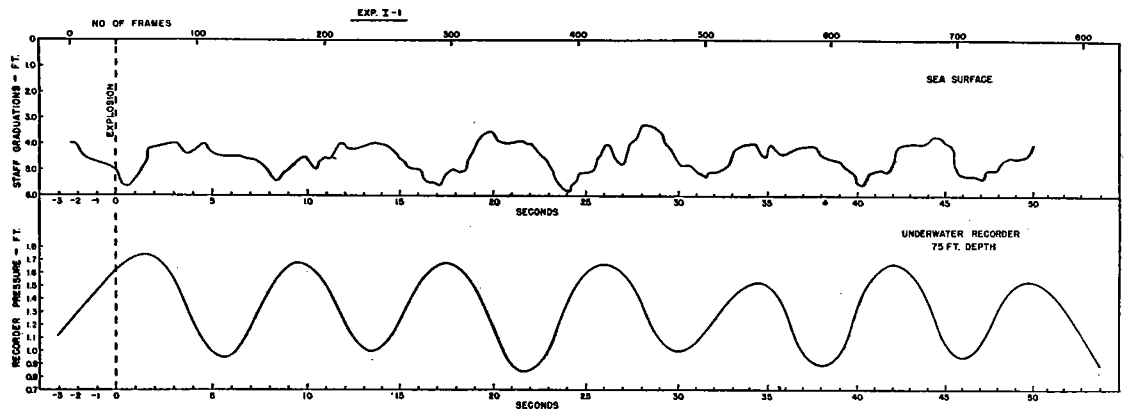

FIG. 6. Sea surface and underwater recorder records for Experiment I-1, off Cuttyhunk, 14 June 1946. Starting time: 10 58 21 EST.

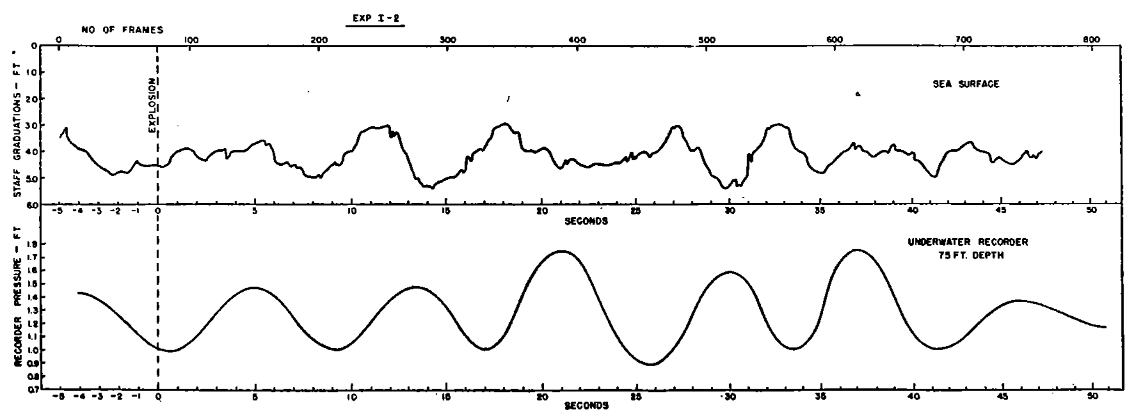

FIG. 7. Sea surface and underwater recorder records for Experiment I-2 off Cuttyhunk, 14 June 1946. Starting time: 11 10 32 EST.

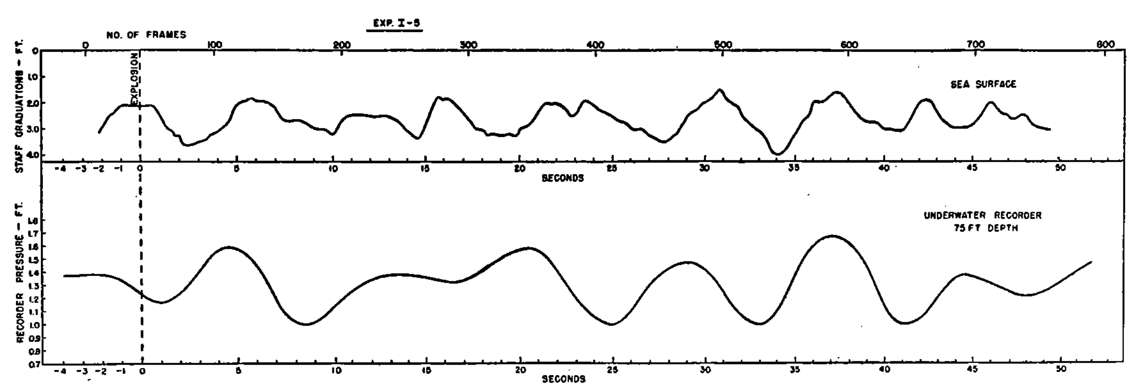

FIG. 8. Sea surface and underwater recorder records for Experiment I-5, off Cuttyhunk, 14 June 1946. Starting time: 11 48 52 EST.

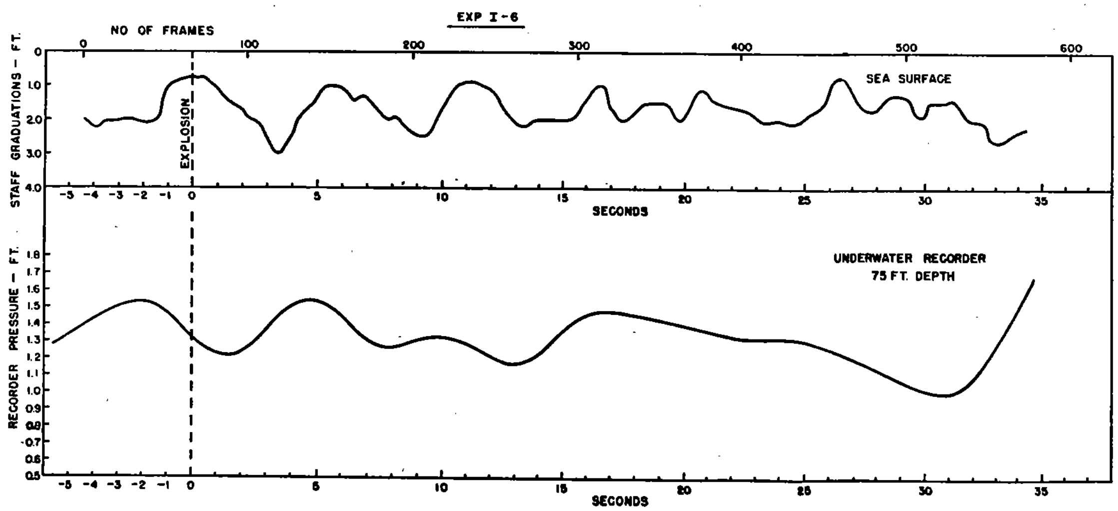

FIG. 9. Sea surface and underwater recorder record for Experiment I-6, off Cuttyhunk, 14 June 1946. Starting time: 12 01 26 EST.

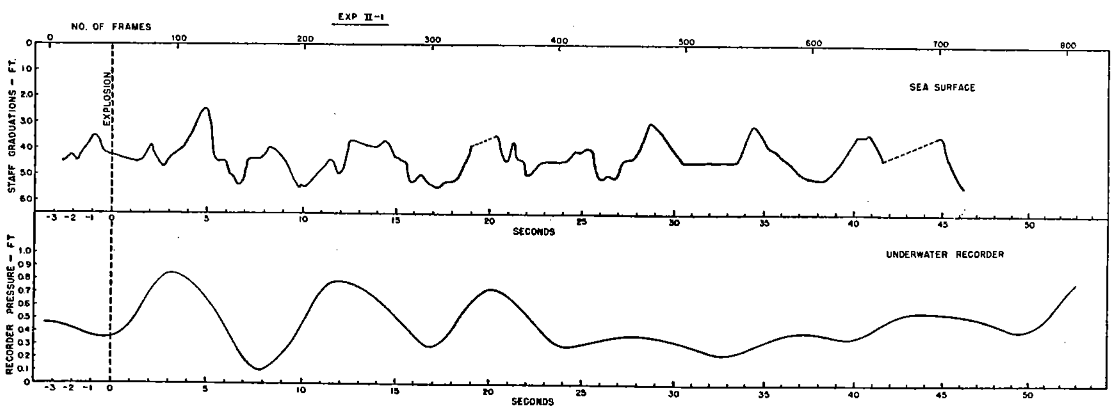

FIG. 10. Sea surface and underwater recorder records for Experiment II-1, off Cuttyhunk, 27 June 1946. Starting time: 10 33 30 EST.

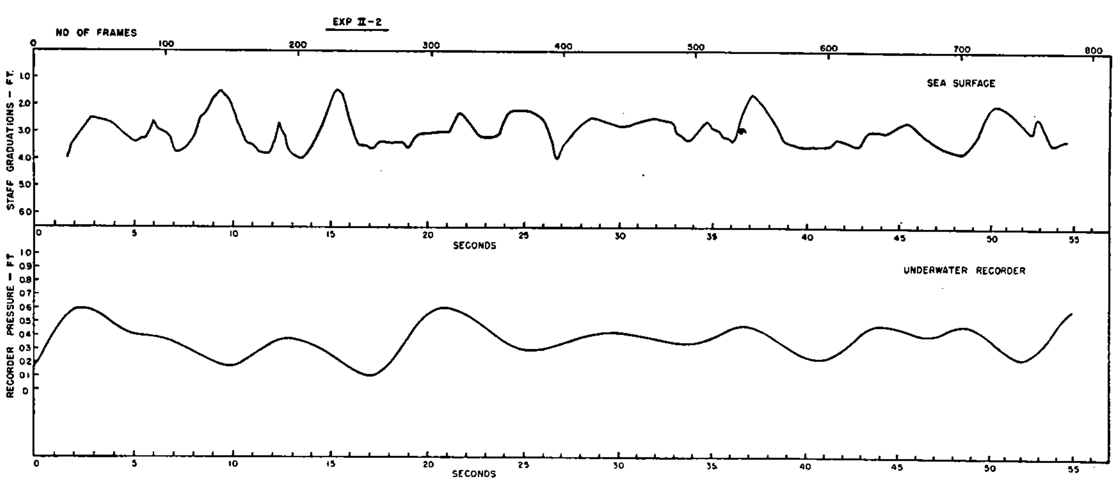

FIG. 11. Sea surface and underwater recorder records for Experiment II-2, off Cuttyhunk, 27 June 1946. Starting time: 10 48 51 EST.

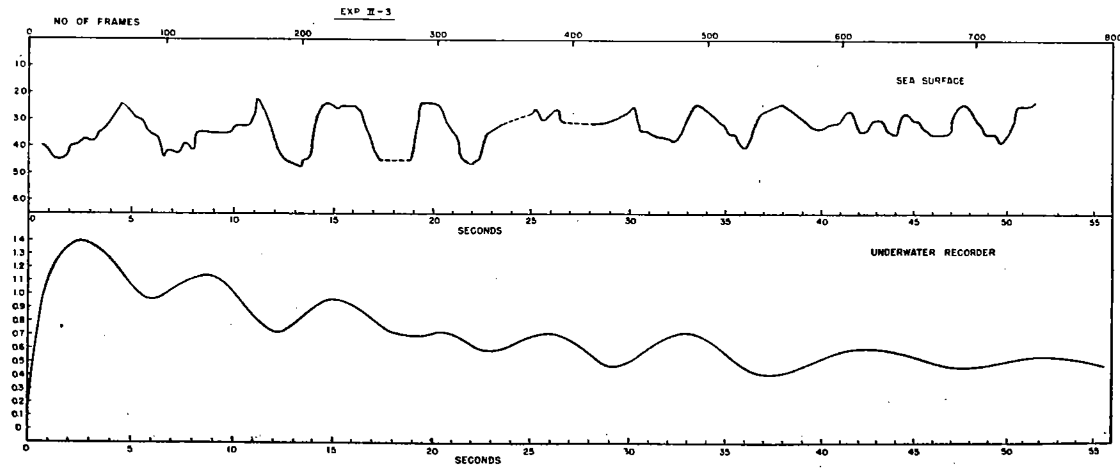

FIG. 12. Sea surface and underwater recorder record for Experiment II-3, off Cuttyhunk, 27 June 1946. Starting time: 10 52 07 EST.

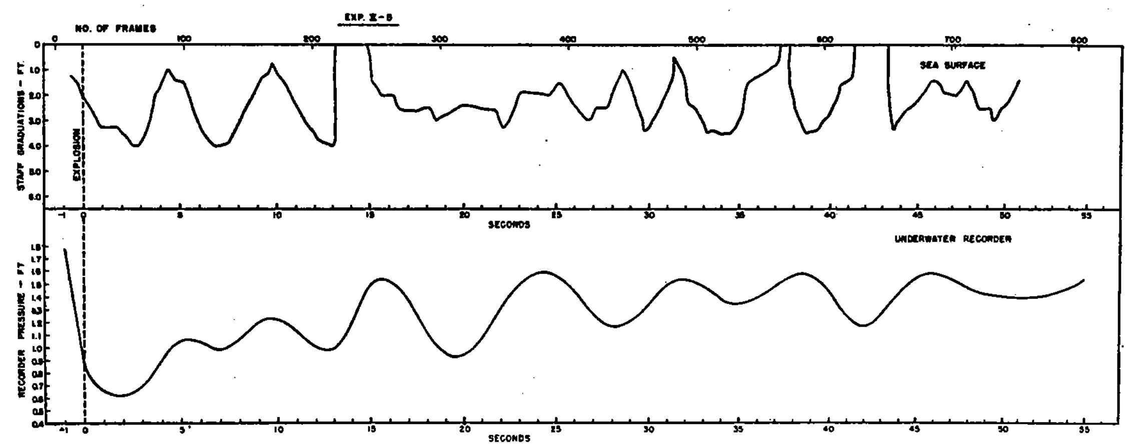

FIG. 13. Sea surface and underwater recorder record for Experiment II-5, off Cuttyhunk, 27 June 1946. Starting time: 11 11 01 EST.

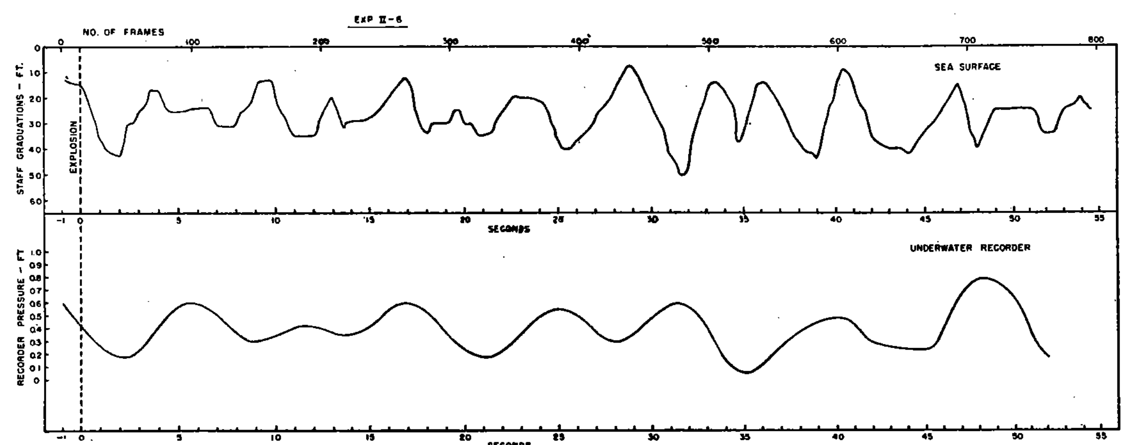

FIG. 14. Sea surface and underwater recorder records for Experiment II-6, off Cuttyhunk, 27 June 1946. Starting time: 11 21 39 EST.

IV. CONDITIONS AT THE SEA SURFACE

1. Interference Patterns

The state of the sea as it affects an observer may be indicated by interference patterns, composed of short time interval averages of wave height and wave period. Such patterns of approximately 50 second intervals, for the two Cuttyhunk and the one Bermuda experiment are given in Table 2. Average values of height and period of the sea surface, for the total duration of each experiment, computed from data in Table 2, are shown in Table 1. Each experiment illustrates characteristics for latter analyses.

It is apparent that sea surface conditions were continuously changing. Those observed off Cuttyhunk Island on June 14 represent the only time the sea surface remained relatively constant during an observational period. On June 27, at this location, both height and average period nearly doubled during the latter ten minutes of the observational period. Similarly, off Bermuda, on October 25, the average sea surface height diminished, and the average period increased during nineteen minutes of observation (Table 1). The rapid sea surface changes appear to result chiefly from variations of the shorter period components set up by local winds; limits of the variability being determined by variations in wind strength and dimensions of local disturbances. The more consistent effects of long period oscillations, induced from sea surface disturbances at greater distances are frequently obscured by local variations.

2. Periodogram and Harmonic Analyses

a. Periodogram Analysis

To obtain supplemental information on individual components making up the sea surface wave patterns, certain of the experiments were subjected to approximate periodogram analyses (5 photo runs of Experiment II) and detailed harmonic analyses (II-1, II-6, V-1, V-2, and V-5).

In carrying out the periodogram analysis[5], scaled values (for each half-second) of the sea surface curves (Figures 10 to 14) were successively arranged in series of horizontal rows, for groups of columns ranging from two to twenty-two. After twenty-one arrangements had been obtained, differences between the highest and lowest sums of each arrangement (Σ_{max}-Σ_{min}) were plotted to form the periodograms of Figure 16.

The dominating periodicities, indicated by variations of Σ_{max}-Σ_{min} are tabulated in Table 3. The limits for periods exceeding 1.5 times the average Σ_{max}-Σ_{min} are between four and eight seconds; the most frequent occurrence being 4.5 to 5.5 seconds.

[5] This method, established for rapid period analysis of economic time series, is described in handbooks of economic statistics.

TABLE 2

SUMMARY

INTERFERENCE PATTERNS FOR SURFACE OBSERVATIONS AND UNDERWATER PRESSURE RECORDS

Experiment No.	I-1	I-2	I-5	I-6	II-1	II-2	II-3	II-5	II-6	V-1	V-2	V-5
Experiment Date	14 June 1946				27 June 1946					25 Oct. 1946		
Time	1058	1111	1149	1201	1034	1049	1052	1112	1122	1453	1457	1512
SEA SURFACE												
Average Height, H_m	1.20	1.17	1.32	1.05	1.0	1.0	1.1	1.9	2.1	1.7	1.4	1.3
Height Range	0.2–2.1	0.4–2.4	0.2–2.4	0.3–2.1	0.3–2.2	0.2–2.5	0.3–2.4	0.1–4.0	0.1–3.6	0.5–4.0	0.5–3.0	0.5–2.0
Av Departure From H_m	0.5	0.6	0.6	0.6	0.6	0.6	0.6	1.15	0.8			
Average Period, T_m	4.33	4.24	4.98	3.81	2.8	3.0	2.7	3.4	3.9	2.5	2.9	3.6
Period Range	1.2–7.2	1.9–9.0	1.5–7.5	1.7–5.8	1.1–6.0	1.7–4.9	0.8–5.6	1.3–6.1	2.5–6.5	0.8–6.3	1.6–4.8	1.5–4.7
Av Departure From T_m	1.8	1.7	1.9	1.6	1.1	0.7	1.3	1.1	0.9			
BOTTOM												
Depth (ft)	75	75	75	75	75	75	75	75	75	120	120	120
Average Height, H_m	0.69	0.58	0.40	0.28	0.35	0.23	0.29	0.39	0.35	0.30	0.33	0.28
Height Range	0.53–0.83	0.37–0.75	0.26–0.67	0.01–0.68	0.08–0.68	0.08–0.50	0.03–1.22	0.15–0.68	0.12–0.55	0.13–0.48	0.11–.58	0.16–40.0
Av Departure From H_m	0.09	0.14	0.06	0.19	0.17	0.13	0.23	0.14	0.11			
Average Period, T_m	8.0	8.3	7.9	7.2	8.3	7.5	7.1	7.0	7.1	9.5	9.3	9.3
Period Range	7.5–8.5	7.0–9.0	7.0–9.0	5.5–10.2	6.8–9.7	5.3–10.5	5.5–10.5	4.3–9.5	5.1–9.0	8.0–13.0	8.0–11.0	9.0–10.0
Av Departure From T_m	0.33	0.58	0.58	1.28	0.80	1.50	1.47	1.12	1.22			
BOTTOM-SURFACE RELATION												
Bottom H_m/Surface H_m	0.575	0.496	0.303	0.267	0.35	0.23	0.26	0.21	0.17	0.18	0.24	0.22
$\Delta P_h/\Delta P_o$ (calc.)	0.444	0.492	0.435	0.329	0.492	0.376	0.314	0.299	0.314	0.377	0.354	0.354
$\Delta P_h/\Delta P_o - H_h/H_o$	−0.131	−0.004	0.102	0.062	0.142	0.146	0.054	0.089	0.144	0.20	0.11	0.13

TABLE 3

Experiment	Periods Exceeding Average Σmax-Σmin	Periods Exceeding 1.5 Average Σmax-Σmin
II-1	4.0, 5.0, 5.5, 7.0, 7.5, 8.0, 9.5, 10.0	5.0, 4.0, 8.0
II-2	3.0, 4.5, 5.0, 5.5. 6.0, 6.5, 7.0, 7.5, 9.5, 11.0	7.0, 6.0
II-3	3.0, 3.5, 4.5, 5.0, 5.5, 7.0, 9.5, 10.0, 11.0	5.0, 4.5
II-5	3.5, 4.0, 4.5, 5.5, 6.5, 7.0, 8.0, 9.5, 11.0	6.0, 6.5, 4.0
II-6	3.5, 4.0, 4.5, 5.0, 6.0, 6.5, 7.0, 8.0, 9.0, 10.0, 10.5	6.0, 6.5, 4.0

Results of sea surface periodogram analysis for Experiment II.

b. Harmonic Analysis

Harmonic analyses were performed on the scaled sea surface data arranged for the above periodogram analyses. As they were already grouped into one-half second intervals it was only necessary to divide the sums of each column by the number of rows to obtain average values. It being assumed that the best representation of variations for selected periods was by means of these average values. Amplitudes and phase values for waves selected from results of the periodogram analysis were computed by the method of least squares[6]. They are tabulated in Table 4.

In this manner, the physical patterns of the sea surface (Experiments II-1, II-6, V-1, V-2, and V-5) are represented by series of sine waves. The probable errors of the amplitudes for surface fluctuations[7] do not exceed 0.02 foot, and recombination of the coefficients (Table 4) reproduce the main features of the observed curves of Figures 10, 14, and 15. However, the individual waves do not necessarily correspond with reality — a doubt which exists in any harmonic analysis of physical data. The principal value of this type of analysis is its convenience in representing data. It is to be noted that the conclusions from this study are not based on the Fourier coefficients, determined as above. The coefficients are used to supplement conclusions arrived at from a direct comparison of surface and bottom pressure patterns.

Both Experiments II and V indicate large time variations of amplitudes for identical waves. Thus, the maximum (Table 4) amplitude change during the forty-nine minute interval of Experiment II was from 0.18 to 0.67 feet for the 6.0 second wave, from 0.47 to 0.96 feet for the 4.5 second wave for the twenty minute interval of Experiment V. The larger variations occurred in amplitudes of the shorter period waves. Table 5 shows that over fifty per cent of the total variations occurred between $T = 4.0$ and $T = 6.0$ seconds for Cuttyhunk, and between $T = 4.5$ and $T = 8.5$ for Bermuda. This apparently results from influence of local generating winds.

[6] For discussion of this procedure, see: Short Period Vertical Oscillations in the Western Basin of the North Atlantic, Papers in Physical Oceanography and Meteorology. Vol. V., No. 2. May 1937.

[7] Probable errors of the Fourier coefficients were computed from the equation $P. E. = e\sqrt{2/N}$, where e is the probable error of the N observations.

Table 4

RESULTS OF HARMONIC ANALYSIS

Surface (o) and Bottom (h) Observations

Experiment		o	h	o	h	o	h	o	h	o	h	o	h	o	h	o	h
	Wave Length, L (ft)	82		105		127		184		214		251		284		328	
	Velocity, c, ft/sec	20.4		23.0		25.5		30.7		33.3		35.8		38.4		41.0	
	Period, T (secs)	4.0		4.5		5.0		6.0		6.5		7.0		7.5		8.0	
	Depth	o	h	o	h	o	h	o	h	o	h	o	h	o	h	o	h
II-1	Amplitude, A (ft)	0.34		0.04		0.42		0.18				0.28		0.40		0.49	
	Phase, α (secs)	0.4		3.9		4.4		1.6				6.1		5.2		5.2	
II-6	Amplitude, A (ft)	0.36		0.34		0.26		0.67	0.05			0.21	0.09	0.10	0.09	0.16	0.07
	Phase, α (secs)	0.8		0.6		4.4		4.6	4.7			0.2	4.6	5.4	2.5	4.2	0.7
	$\alpha_h - \alpha_o$							0.1				4.4		−2.9		−3.5	
V-1	Amplitude, A (ft)			0.58	0.138					0.26	0.019			0.41	0.013		
	Phase, α (secs)			0.5	3.4					0.2	5.2			5.8	0.3		
	$\alpha_h - \alpha_o$			2.9						5.0				2.0			
V-2	Amplitude, A (ft)			0.47	0.014					0.25	0.009			0.11	0.032		
	Phase, α (secs)			0.9	2.9					4.9	3.0			1.2	2.5		
	$\alpha_h - \alpha_o$			2.0						4.6				1.3			
V-5	Amplitude, A (ft)			0.96	0.003					0.29	0.009			0.24	0.015		
	Phase, α (secs)			2.1	0.1					4.9	2.7			1.4	2.5		
	$\alpha_h - \alpha_o$			2.5						4.3				1.1			

Experiment		o	h	o	h	o	h	o	h	o	h	o	h	o	h		
	Wave Length, L (ft)	365		415		461		512		536		618		736			
	Velocity, c, ft/sec	43.5		46.0		48.6		51.2		53.7		56.4		61.5			
	Period, T, (secs)	8.5		9.0		9.5		10.0		10.5		11.0		12.0			
	Depth	o	h	o	h	o	h	o	h	o	h	o	h	o	h		
II-1	Amplitude, A (ft)	0.10		0.21		0.30		0.22									
	Phase, α (secs)	6.1		4.5		3.9		1.2									
II-6	Amplitude, A (ft)	0.11		0.04	0.07	0.17	0.01	0.25									
	Phase, α (secs)	0.9		8.5	5.5	8.5	5.7	7.6									
	$\alpha_h - \alpha_o$			−3.0		−2.8											
V-1	Amplitude, A (ft)	0.43	0.045	0.34	0.018	0.21	0.025	0.15	0.052	0.13	0.023	0.34	0.011	0.08	0.018		
	Phase, α (secs)	4.1	4.7	3.5	1.4	2.4	7.9	1.6	7.8	2.2	8.7	1.9	3.4	0.4	3.7		
	$\alpha_h - \alpha_o$	0.6		6.9		5.5		6.2		6.5		1.5		3.3			
V-2	Amplitude, A (ft)	0.09	0.018	0.15	0.058	0.06	0.001	0.06	0.077	0.15	0.024	0.07	0.028	0.11	0.018		
	Phase, α (secs)	7.6	0.0	6.6	3.2	6.1	2.4	8.5	4.4	9.1	4.4	7.8	9.6	7.2	11.2		
	$\alpha_h - \alpha_o$	0.9		5.6		5.8		5.9		5.8		1.8		4.0			
V-5	Amplitude, A (ft)	0.11	0.066	0.09	0.003	0.09	0.090	0.20	0.056	0.27	0.031	0.13	0.033	0.26	0.007		
	Phase, α (secs)	7.5	7.6	6.4	3.8	0.8	4.1	0.0	6.3	9.0	4.4	8.6	10.7	8.6	0.8		
	$\alpha_h - \alpha_o$	0.1		6.4		6.2		6.3		5.9		2.1		4.2			

TABLE 5

PERIOD SECONDS	EXPERIMENT II	EXPERIMENT V
4.0	1.0%	
4.5	14.9%	21.3%
5.0	8.0%	
6.0	24.4%	
6.5		1.7%
7.0	3.5%	
7.5	14.9%	13.0%
8.0	16.4%	
8.5	0.5%	14.8%
9.0	8.5%	10.9%
9.5	6.5%	6.5%
10.0	1.5%	6.1%
10.5		6.1%
11.0		11.7%
12.0		7.8%

Relative time changes in amplitudes of individual wave periods at the surface; Experiment II, 1034 to 1122, 27 June 1946; Experiment V, 1453 to 1512, 25 October 1946.

V. CONDITIONS NEAR THE SEA BOTTOM

1. INTERFERENCE PATTERNS

Interference patterns of bottom pressure fluctuations, computed from scaled values of the underwater records for the Cuttyhunk and Bermuda experiments are tabulated in Table 2, and average values are shown in Table 6[8]. The longer average periods of the Bermuda record are influenced by the greater depth of instrumentation; their relationships to simultaneous surface wave patterns are discussed later.

TABLE 6

	EXPERIMENT I	EXPERIMENT II	EXPERIMENT V
Location	Cuttyhunk	Cuttyhunk	Bermuda
Depth	75 ft.	75 ft.	120 ft.
Average Height	0.49	0.32	0.30
Average Period	7.85	7.40	9.37

Average values of bottom interference patterns computed from data in Table 2.

2. PERIODOGRAM AND HARMONIC ANALYSES

a. Periodogram Analysis

The method of periodogram analysis of bottom pressure records differed from that used for surface records (see Footnote 6) in that the analyses were made mechanically on the "Ocean Wave Analyzer"[9]. For this analysis, a suitable length of record (Figure 5)

[8] Underwater pressure records were evaluated from calibration data provided by Mr. Klebba. Average interference patterns were computed for the surface from measurements of crest heights and time intervals between crests.
[9] Descriptions of the "Ocean Wave Analyzer" are on file at this Institution.

is wound around the twenty-inch wheel of the analyzer which is then rotated to a high speed. As the wheel slowly decelerates, waves of successively shorter periods on the tape pattern are brought in tune with an electrical circuit. The output for each frequency (between 4 and 40 seconds) is registered by a pen moving over a chart (Figures 17 and 18), the horizontal movement of which is synchronized with the decelerating speed of the analyzer wheel. As an example of the selective tuning, when the wheel rotates at 240 RPM, forty second waves on the tape are selected and at 24 RPM, four second waves on the tape are brought in tune with the analyzer circuit. The relative output for each frequency is recorded as a displacement of the ordinate as the chart is moved to proper period position by a speed indicator.

In view of this type of analysis, as compared with somewhat similar statistical procedures, and because distances of the ordinates from the base line indicate only relative significance of the periods in the frequency band, it is here referred to as a mechanical periodogram analysis.

Comparative studies by the designer indicate that with present adjustments, a maximum accuracy of plus or minus one-half second is obtained for the period band between six and twenty seconds; in the 30 second region this diminishes to plus or minus two seconds.

In Table 7, the limits of the principal spectrum band, and the three principal periods, from the analyzer records, are tabulated for Experiments II and V. The average period of the bottom interference pattern (Table 2) lies within the principal band.

TABLE 7

	Cuttyhunk, Exp. II			Bermuda, Exp. V	
Photo Run	1, 2, 3	5	6	1	2 5
Analysis	II-A	II-B	II-C	B-12-c	B-12-d
Principal Spectrum	7.0–10.0	7.0–10.0	7.0–10.0	7.5–15.0	7.5–14.0
Principal Period	9.5	8.5	7.5	10.0	10.5
Second Period	7.5	9.0	8.0	9.5	11.0
Third Period	8.5	8.0	9.0	11.0	10.0

Scaled data from mechanical periodogram analysis of indicated bottom pressure records (Figures 17 and 18).

b. *Harmonic Analysis*

Data for the harmonic analysis of bottom pressure records of Experiments II-6, V-1, V-2, and V-5 were obtained by scaling approximately one minute sections of the records corresponding with the times of surface observations. Replottings of data for Experiments I and II (in one-half second intervals) are illustrated by Figures 6 to 14. These data are also used later for determining quantitative relationships between bottom and surface pressure fluctuations (Figure 19).

The Fourier amplitudes[10] computed by the method of least squares have a probable error of approximately 0.01 feet (Table 4). The relative magnitudes of the amplitudes (Table 8) confirm the sequences brought out by the periodogram analyses (Figures 17 and 18), and tabulated in Table 7. Thus, in three of the four cases, wave periods having

[10] See Footnotes 6 and 7.

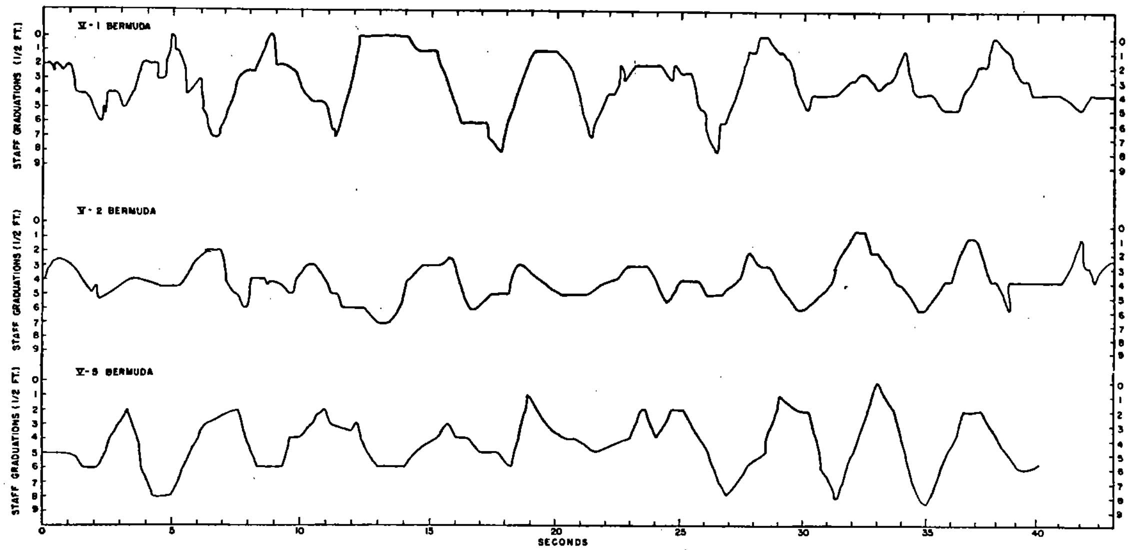

FIG. 15. Sea surface records for Experiment V, photo runs, 1, 2, and 5, off Bermuda, 25 October 1946. Starting times 14 53, 14 57 and 15 12 EST, respectively.

maximum Fourier amplitudes were identical with those shown to have maximum amplitudes by the mechanical analysis. This agreement is significant in that results of the mechanical wave analyses appear to (within limits indicated above) indicate individual waves dominating bottom pressure records.

TABLE 8

	EXPER. II-6		EXPER. V-1		EXPER. V-2		EXPER. V-5	
	T (SEC)	A (FT)	T (SEC)	A (FT)	T (SEC)	A (FT)	T (SEC)	A (FT)
First Period.......	7.5	0.09	10.0	0.052	10.0	0.077	9.5	0.090
Second Period.....	7.0	0.09	8.5	0.045	9.0	0.058	8.5	0.066
Third Period......	8.0	0.07	9.5	0.025	7.5	0.032	10.0	0.056

First three principal periods (T) and amplitudes (A) by harmonic analysis of bottom pressure records.

VI. RELATIONSHIP OF BOTTOM TO SURFACE PRESSURE FLUCTUATIONS

1. THEORETICAL CONSIDERATIONS

Considerations of wave motion in an incompressible fluid of constant density, ρ, where the depth, h, is greater than the wave length, λ, provide the theoretical basis for investigations of ocean waves. In the following, it is assumed that motion has started from rest by natural forces; the viscosity influence is neglected. Since the motion is irrotational, a velocity potential, ϕ, is assumed. The usual boundary conditions of hydrodynamics, including the Laplace equation of continuity, are to be satisfied.

In the following, the z axis is taken vertically upwards, the xy plane in the undisturbed free surface, and motion is assumed in the unlimited x direction. Thus, a solution to the Laplace equation of continuity:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{1}$$

representing a progressive wave with velocity c, is:

$$\phi = (Ae^{kz} + Be^{-kz}) \operatorname{Cos} k\,(x\text{-}ct) \tag{2}$$

At the bottom ($z = -h$) the condition to be satisfied is:

$$\frac{\partial\phi}{\partial z} = 0 \tag{3}$$

Thus

$$Ae^{-kh} - Be^{kh} = 0 \tag{4}$$

and, let

$$Ae^{-kh} = Be^{kh} = R/2 \tag{5}$$

Hence

$$\phi = R \operatorname{Cosh} k\,(z + h) \operatorname{Cos} k\,(x\text{-}ct) \tag{6}$$

a. *The Surface Wave Velocity*

The condition at the free sea surface ($z = 0$) is:

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0 \tag{7}$$

Hence, from (6)

$$c^2 k \operatorname{Cosh} kh \operatorname{Cos} k\,(x\text{-}ct) = g \operatorname{Sinh} kh \tag{8}$$

and

$$c^2 = \frac{g}{k} \operatorname{Tanh} kh \tag{9}$$

Since

$$k = \frac{2\pi}{\lambda} \tag{10}$$

we have

$$c^2 = \frac{g\lambda}{2\pi} \operatorname{Tanh} \frac{2\pi h}{\lambda} \tag{11}$$

If the water is very shallow, so that

$$\operatorname{Tanh} \frac{2\pi h}{\lambda} = \frac{2\pi h}{\lambda}$$

then

$$c^2 = gh \tag{12}$$

In this case we have as examples of $\frac{h}{\lambda}$ limits:

$\frac{2\pi h}{\lambda}$	$\operatorname{Tanh} \frac{2\pi h}{\lambda}$	$\frac{h}{\lambda}$
0.050	0.04996	0.0080
0.100	0.09967	0.0160
0.250	0.24492	0.0398
0.500	0.46212	0.0796

If the water is sufficiently deep so that

$$\text{Tanh}\,\frac{2\pi h}{\lambda} = 1$$

then

$$c^2 = \frac{g\lambda}{2\pi} \tag{13}$$

In this case $\frac{h}{\lambda}$ limits are as follows:

$\frac{2\pi h}{\lambda}$	$\text{Tanh}\,\frac{2\pi h}{\lambda}$	$\frac{h}{\lambda}$
1.832	0.950	0.292
2.647	0.990	0.421
3.800	0.999	0.605

Other relationships between velocity, c, wave length, λ, period, T, wave number, k, and frequency, σ, of deep water waves, are:

$$c = \sqrt{\frac{g\lambda}{2\pi}} = \frac{\lambda}{T} = \frac{gT}{2\pi}, \tag{14}$$

$$\lambda = \frac{2\pi}{g}c^2 = \frac{g}{2\pi}T^2, \tag{15}$$

$$T = \sqrt{\frac{2\pi}{g}\lambda} = \frac{2\pi}{g}c, \tag{16}$$

$$k = \frac{2\pi}{\lambda} \tag{17}$$

and

$$\sigma = \frac{2\pi}{T} \tag{18}$$

b. Pressure Fluctuations at Surface and Bottom

The elevation of water, ζ, at the free surface above the point x, y, o is:

$$\zeta = \frac{1}{g}\frac{\partial\phi}{\partial t} \tag{19}$$

Hence, from (6)

$$\zeta = \frac{kcr}{g}\,\text{Cosh}\,kh\,\text{Sin}\,k\,(x\text{-}ct) \tag{20}$$

Letting a be the amplitude of ζ,

$$a = \frac{kcr}{g}\,\text{Cosh}\,kh, \tag{21}$$

then

$$\zeta = a\,\text{Sin}\,k\,(x\text{-}ct) \tag{22}$$

thence from (6)

$$\phi = \frac{ga}{kc} \frac{\text{Cosh } k\,(z + h)}{\text{Cosh } kh} \text{Cos } k\,(x\text{-}ct) \tag{23}$$

The pressure, P, at any depth is:

$$P = \rho g\,(\zeta - z) \tag{24}$$

and the pressure fluctuation, ΔP, at any depth is:

$$\Delta P = \rho ga \frac{\text{Cosh } k\,(z + h)}{\text{Cosh } kh} \text{Sin } k\,(x\text{-}ct) \tag{25}$$

At the bottom, $z = -h$, and

$$\text{Cosh } k\,(z + h) = 1 \tag{26}$$

Hence the pressure fluctuation at the bottom, ΔP_h is

$$\Delta P_h = \rho ga \frac{\text{Sin } k\,(x\text{-}ct)}{\text{Cosh } kh} \tag{27}$$

At the surface, $z = 0$, and

$$\Delta P_s = \rho ga \text{ Sin } k\,(x\text{-}ct) \tag{28}$$

Thus, the ratio of instantaneous pressure fluctuations at the bottom to those at the surface[11] is

$$\frac{\Delta P_h}{\Delta P_s} = \frac{1}{\text{Cosh } kh} \tag{29}$$

2. Bottom-surface Relations of Observed Fluctuations

It may be noted here that some opposition has been offered to the following method of analysis in that it does not fall entirely within the frame work of customary shallow water wave theory. In the case of certain special wave analyses this criticism is reasonable. However, where we deal with complex wave patterns, composed of both shallow and deep water phenomena, and where the effects of wave transformation are imperfectly known and require empirical correction in each special case, it has appeared advisable to treat the problem in the most direct manner; namely, to approach it from established relationships for deep water waves. Hence, any alterations resulting from shallow water transformations are included in the deduced empirical factors. The very consistency of the results obtained, appear to justify the method of approach in that they serve as a means to an end. In the future, when more observational data is available, and when the phenomena of shallow water wave transformations are better understood, some refinement to this approach may be desirable.

In connection with the above, a comparison of surface and bottom interference patterns (Table 2) shows that at Cuttyhunk the bottom wave period ranged 1.6 to 3.0 times that at the surface and at Bermuda it was 2.6 to 3.8 times the surface. Hence in directly comparing surface and bottom wave profiles, short period waves play a prominent role in surface displacements which probably are not reflected in pressure changes at instrument depths for either locality.

[11] This relationship has been extensively used by earlier investigators. It is derived in Horace Lamb's Hydrodynamics.

Surface wave heights (ΔH_s) and wave periods (ΔT_s) were obtained directly by observation. Bottom pressure recordings were evaluated in terms of bottom wave heights (ΔH_h) and bottom wave periods (ΔT_h) from instrument factors. Comparison of surface and bottom wave profiles at Cuttyhunk and Bermuda (Figures 6 to 15) permitted good identification of more than 50 prominent pairs of wave crests. From these, the ratios, $\Delta H_h/\Delta H_s$ were computed. To this series of ratios were added those computed from the interference patterns of Table 2.

The ratios $\Delta H_h/\Delta H_s$ are plotted against bottom wave periods (ΔT_h) for comparison with theoretical values, from Equation 29, in Figure 19. It is apparent that the computed ratios are generally lower than to be expected from theory, as expressed by Equation 29. This being the case, surface wave height computed from bottom pressure records by application of Equation 29 will be too low.

It is essential to examine the extent of the departures of ratios computed from observation, from that given by theory (Equation 29), and, if possible, to establish from our experiments an empirical factor which when applied to bottom pressure records gives reasonable estimations of the causative sea surface wave patterns. Measurements of underwater pressure fluctuations are highly significant to research on sea surface waves, in that they should provide reasonably accurate state of the sea estimations. In fact, estimations of the state of the sea surface from bottom pressure fluctuations are to be preferred over those of direct observation, in that effects of very short period waves, which confuse the surface picture, are eliminated. However, application of the hyperbolic cosine relation of Equation 29

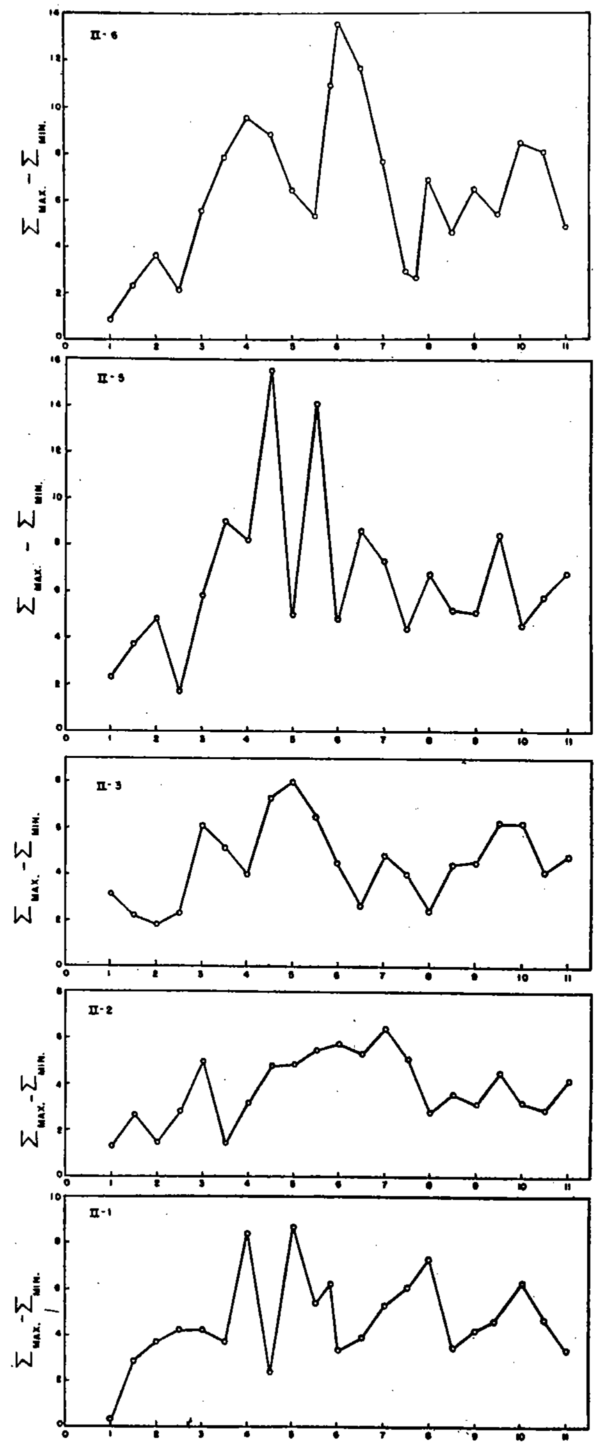

FIG. 16. Results of periodogram analysis of Experiment II. (see text).

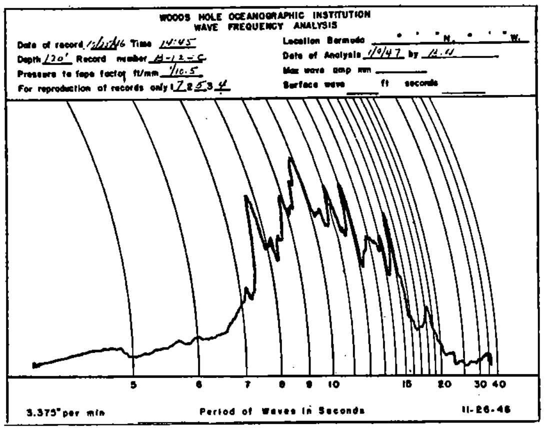

FIG. 17. Periodogram analysis of underwater pressure record (75 feet depth). Experiment No. II, 27 July 1946, 1033-1054 EST, Cuttyhunk. Analysis includes time of surface photo runs II-1, II-2, and II-3.

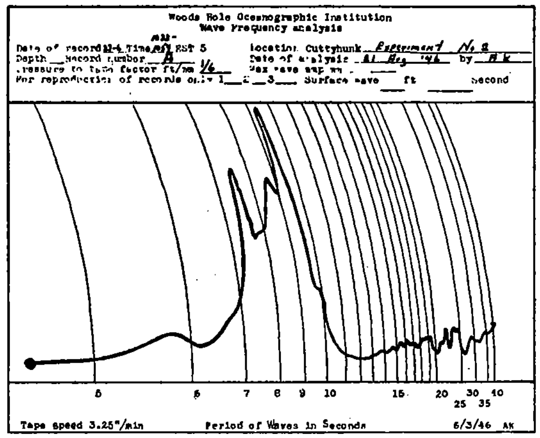

FIG. 18. Periodogram analysis of underwater pressure record (120 feet depth). Experiment No. V, 25 October 1946, 1340-1358 EST, Bermuda. Analysis includes time of surface photo run V-1.

to underwater pressure records needs be adjusted if states of sea surface are to be estimated with reasonable accuracy.

The method used in arriving at a conclusion is as follows: From the sixty-four pairs of identifiable bottom and surface observations, statistics were computed to show relations of observed ($\Delta H_h/\Delta H_s$) to theoretical values ($\Delta P_h/\Delta P_s$). The average differences between these values ($\Delta P_h/\Delta P_s - \Delta H_h/\Delta H_s$), and the average ratios $\left(\frac{\Delta P_h}{\Delta P_s}\Big/\frac{\Delta H_h}{\Delta H_s}\right)$ were found to be nearly constant for all experiments. Hence, it was considered justifiable to combine them and use the mean values of $\frac{\Delta P_h}{\Delta P_s} - \frac{\Delta H_h}{\Delta H_s} = 0.11 \pm 0.007$ and of $\frac{\Delta P_h}{\Delta P_s}\Big/\frac{\Delta H_h}{\Delta H_s} = 1.35 \pm 0.037$ to characterize the data under consideration. The dispersions of arrangements of data, and the associated probable errors, do not contradict this procedure. The consistency of results for the two locations (Cuttyhunk and Bermuda), approximately 750 miles apart, characterized by different hydrographic situations, at bottom depths of 75 and 120 feet and with observation times four months apart, suggests that the statistics of Table 9 are empirically significant.

TABLE 9

EXPERIMENT	RANGE $\frac{\Delta P_h}{\Delta P_s} - \frac{\Delta H_h}{\Delta H_s}$	AVERAGE $\frac{\Delta P_h}{\Delta P_s} - \frac{\Delta H_h}{\Delta H_s}$	AVERAGE $\frac{\Delta P_h}{\Delta P_s}\Big/\frac{\Delta H_h}{\Delta H_s}$
I and II (I. P.)	−0.13 to +0.15	0.11	1.31
I and II (o.)	−0.07 to +0.28	0.12	1.37
V (I. P. and o.)	−0.02 to +0.20	0.10	1.37
Average		0.11	1.35
Standard Deviation		0.086	0.407
Probable Error		0.007	0.037

Summary of bottom-surface fluctuation ratios ($\Delta H_h/\Delta H_s$) from interference patterns (I. P.) and from direct comparison of observations (o.) compared with theoretical bottom-surface ratios from Equation 29 ($\Delta P_h/\Delta P_s = 1/\text{Cosh } kh$).

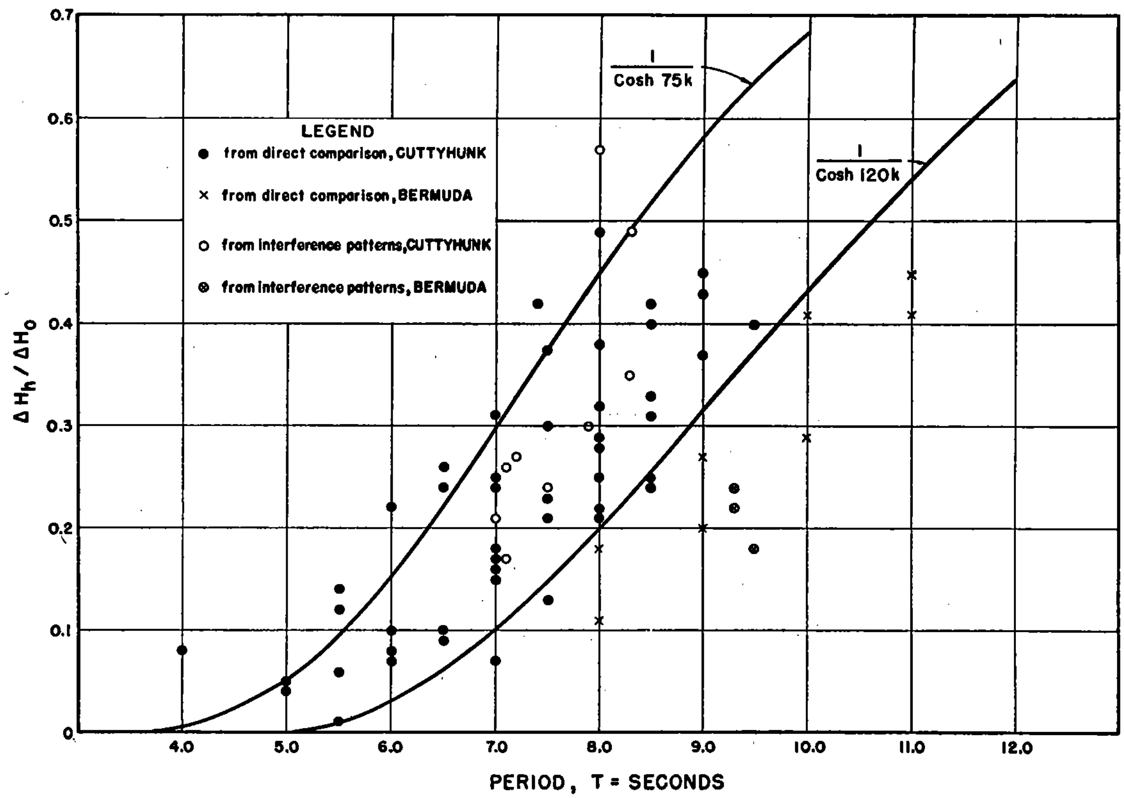

FIG. 19. Ratios of bottom to surface pressure fluctuations ($\Delta H_h/\Delta H_o$) as indicated. Theoretical curves for Cuttyhunk (75 foot depth) and Bermuda (120 foot depth), without shallow water correction.

Surface wave heights entering into the present analysis ranged from 0.5 to 3.0 feet, sixty-eight per cent of which were between 1.1 and 2.0 feet. A progressive breakdown of the relation: $\frac{\Delta P_h}{\Delta P_s} / \frac{\Delta H_h}{\Delta H_s}$, indicates that as surface wave heights (ΔH_s) increase, theoretical and observed ratios become more divergent. This is brought out in Table 10 where $\frac{\Delta P_h}{\Delta P_s} / \frac{\Delta H_h}{\Delta H_s}$ progressively increased from 1.22 for surface wave heights of 0.1 to 1.0 feet to 1.44 for surface waves of 2.1 to 3.0 feet. Although the results are suggestive, they are not adequate, as the differences barely exceed the probable errors of the computation. Additional field and laboratory research is desirable.

TABLE 10

SURFACE WAVE HEIGHT (FEET)	PER CENT OF OBSERVATIONS	$\frac{\Delta P_h}{\Delta P_s} / \frac{\Delta H_h}{\Delta H_s}$	PROBABLE ERROR
0.1 to 1.0	15	1.22	0.096
1.1 to 2.0	68	1.37	0.045
2.1 to 3.0	17	1.44	0.089

Percentage wave height distribution for sixty-four wave crests analyzed and associated values of $\frac{\Delta P_h}{\Delta P_s} / \frac{\Delta H_h}{\Delta H_s}$

3. The Empirical Relationship of Observed to Theoretical Bottom-surface Fluctuations

On the basis of data described above for Cuttyhunk and Bermuda, the theoretical ratio of bottom to surface pressure fluctuations has an average value of 1.35 ± 0.04 times that computed from observations. The evidence indicates this value to be empirically significant to the data under consideration. Thus

$$\frac{\Delta P_h}{\Delta P_s} = 1.35\,\frac{\Delta H_h}{\Delta H_s} \tag{30}$$

or

$$\frac{\Delta H_h}{\Delta H_s} = 0.741\,\frac{\Delta P_h}{\Delta P_s} \tag{31}$$

and

$$\Delta H_s = 1.35\ \Delta H_h \Big/ \frac{\Delta P_h}{\Delta P_s} \tag{32}$$

From Equation 29, we have as the theoretical relationship

$$\frac{\Delta P_h}{\Delta P_s} = 1/\text{Cosh kh}$$

Thus substituting,

$$\Delta H_s = 1.35\ \Delta H_h\ \text{Cosh kh} \tag{33}$$

where h is the depth of the pressure measuring instrument close to the bottom, and k is the wave number.

Equation 33 is used in preference to Equation 29 for computing average and individual wave heights at the sea surface from underwater pressure records at Cuttyhunk and Bermuda. The suggested technique for evaluation is described later. It is particularly applicable for surface wave heights up to three feet; for higher waves the analysis suggests the empirical factor of 1.35 should be increased.

4. Bottom-surface Relations of Fourier Amplitudes of Component Waves

Previous discussion has been concerned with comparison of factors derived from the complex surface and bottom wave patterns. Some idea of the bottom to surface relations of individual wave components, making up the patterns, may possibly be obtained from examination of their Fourier amplitudes (Chapter IV and V). However, since the physical reality of Fourier coefficients is subject to doubt, little is to be gained, for the present, by more than a cursory examination. The ratios are widely scattered and in some cases exceed unity. This, in part, is caused by the probable error effect which becomes relatively large for the small amplitudes.

To arrange the data in suitable fashion, surface (ΔA_s) and bottom (ΔA_h) Fourier amplitudes (of Table 4) were adjusted by either adding or subtracting the probable errors so that the computed ratios ($\Delta A_h/\Delta A_s$) were closest to that required by theory

(Equation 33). The results are plotted in Figure 20; about sixty per cent of the Bermuda ratios follow the requirements of the modified hyperbolic cosine relation (Equation 33). The remainder scatter so as to require two to three times the probable error adjustment to bring them into line.

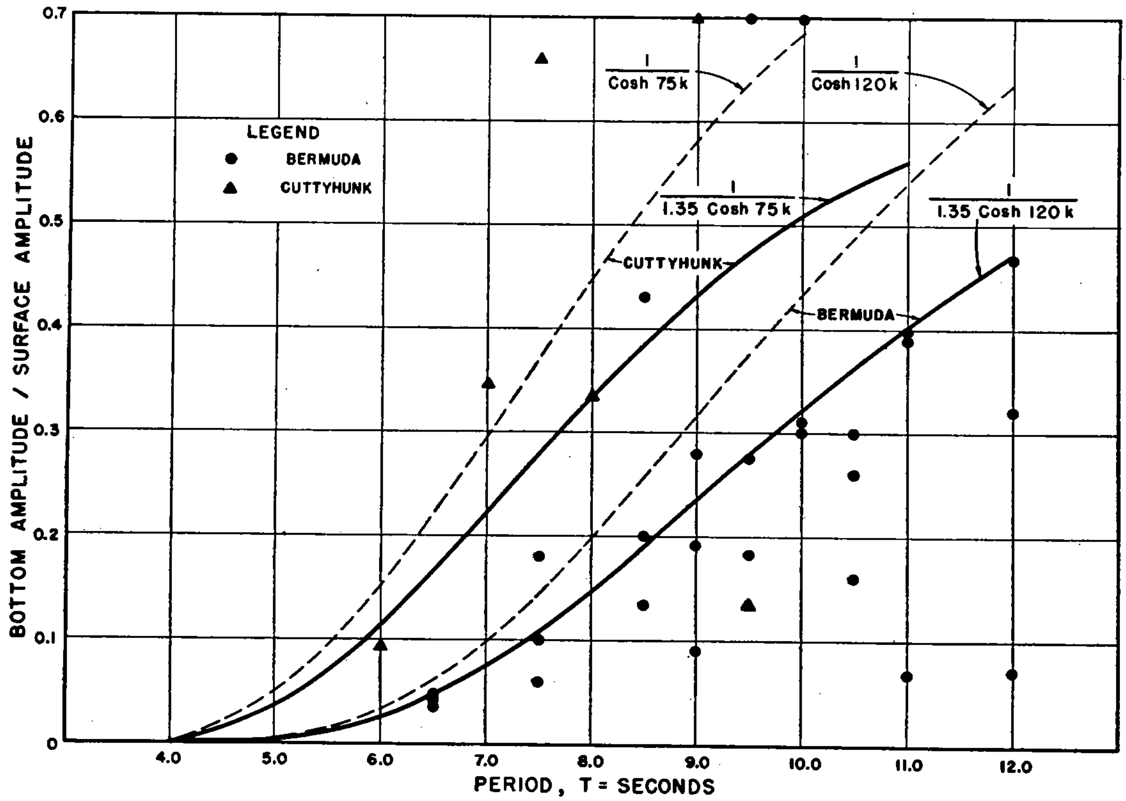

FIG. 20. Ratios of adjusted bottom-surface Fourier amplitudes compared with theoretical and empirical values for Cuttyhunk and Bermuda.

It is apparent from Figure 20 that departures of Fourier ratios from the theory are greater for higher wave periods. Thus, between periods of 6 and 9 seconds, ratios follow the 1/1.35 cosh 120 k relationship to a good degree of approximation; but above 9 seconds departures are progressively greater, and in a direction indicating abnormally high surface wave heights. The situation may possibly be linked with the increased steepness of the component waves as they advance into shallow water.

At both Cuttyhunk (depth 75 feet) and Bermuda (depth 120 feet) some change in steepness occurs as the waves advance into the shallower water of instrument depth. The deep water wave steepness, δ_0, defined as the ratio of wave height, H_0, to wave length λ_0, theoretically increases after certain critical depths are reached. The change in steepness, designated as the ratio of shallow water steepness, δ, to deep water steepness, δ_0, is

$$\frac{\delta}{\delta_o} = \frac{H/H_o}{\lambda/\lambda_o} \tag{34}$$

Indications of theoretical changes in steepness for individual component waves of the periods observed at Cuttyhunk and Bermuda are given by the ratios of λ/λ_0 in Table 11.[12] They provide a partial explanation of the departures of Fourier ratios from the theoretical values of Figure 20.

TABLE 11

		CUTTYHUNK		BERMUDA	
PERIOD	λ_0	d/λ_0	λ/λ_0	d/λ_0	λ/λ_0
6.0	184	0.41	0.99	0.65	1.00
6.5	214			0.56	1.00
7.0	251	0.30	0.96	0.48	1.00
7.5	284	0.26	0.94	0.42	0.99
8.0	328	0.23	0.92	0.37	0.98
8.5	365			0.33	0.97
9.0	415	0.18	0.86	0.29	0.96
9.5	461	0.16	0.84	0.26	0.94
10.0	512			0.23	0.92
10.5	536			0.22	0.91
11.0	618			0.19	0.88
12.0	736			0.16	0.84

Ratios of shallow (λ) to deep water (λ_0) wave lengths of individual component waves at instrument depth for Cuttyhunk (d = 75 ft) and Bermuda (d = 120 ft).

5. Bottom-surface Phase Angle Relations of Fourier Coefficients

Due to the fact that observing locations on the sea surface were not directly over the underwater pressure records (Chapter III), differences in the surface and bottom phase angles of Fourier coefficients were used to locate directions of the individual wave fronts which, it is assumed, have combined to produce surface and bottom interference patterns. Here again the question of physical reality of representation by Fourier series enters, and although the following may be fictitious, it introduces a method which may become useful in determining directionality of wave fronts.

The three series available from Bermuda (V-1, V-2, V-5) show remarkably constant differences of bottom and surface phase angles (α_h-α_s) regardless of the variability in occurrence time of the maxima (Table 4). The computation of directionality for Bermuda is based on the average α_h-α_s of the three experiments; that for Cuttyhunk on one single experiment, II-6.

The geometric and geographic properties of the situation determine the direction of the advancing wave fronts with reference to a horizontal line connecting the surface and bottom observation points. The horizontal distance, D, between these two points was less than one wave length. Hence, the angle β, between the direction of the advancing wave front and the connecting line is

$$\beta = \mathrm{Cos}^{-1} \frac{\alpha_h - \alpha_s}{D} \tag{35}$$

[12] Data from Sverdrup and Munk: Breakers and Surf. U. S. Navy Department, H. O. No. 234, 1944.

The results showing directions of component wave fronts for Cuttyhunk and Bermuda, at times of the experiments, are illustrated by Figures 1 and 2.

VII. THE ESTIMATION OF AVERAGE STATES OF THE SEA SURFACE FROM UNDERWATER PRESSURE RECORDS

In the foregoing analysis, the preliminary supporting data are given in some detail. This is considered essential to the central problem, in that future investigations will have available the background of experimental data for comparison with results from other areas. It is considered too early to generalize the results of this investigation, for the present they are considered applicable only to the Woods Hole and Bermuda localities.

The central result brought out by the present investigation is that, for the Woods Hole and Bermuda localities, wave heights at the sea surface (ΔH_s) are related to those near the sea bottom (ΔH_h) by:

$$\Delta H_s = 1.35 \ \Delta H_h \text{ Cosh } kh$$

In the above, h, is the depth of the water column and k is the wave number ($k = (2\pi)^2/gT^2$). The empirical factor of 1.35, has been determined from controlled observational data for surface wave heights up to three feet.

This result establishes a valid basis for action, in that a means is provided for obtaining improved computations of sea surface conditions from underwater pressure records. For the moment, this is known to apply only to the localities concerned. Modifications, of the empirical relationship, and its extension to other areas, will possibly result from repeated experimentation under a variety of conditions.

When the problem is to compute average wave heights and periods of the overlying physical sea surface from underwater pressure records, spacing of records will depend on specific requirements. The present procedure at this Institution is to operate the underwater pressure recorders for twenty minutes out of every two, four or six hours; a method established with reference to development of the instrument.

To compute average wave heights of the sea surface, wave heights are scaled from bottom pressure records, and converted to feet of sea water pressure by the instrument factor[13]. An average height is then computed and the average period taken as the average time interval between successive crests. Noting average crest height at the bottom by ΔH_h and using the average period to compute the wave number k, the average wave height at the sea surface, ΔH_s is computed from Equation 33.

The average period of the bottom wave crests, determined by direct measurements of the pressure records, will represent an upper limit for the average sea surface period. Generally it is to be expected this average bottom wave period will be two or more times that at the surface, depending on depth of instrumentation. From an operational standpoint, this is important, since for identical wave heights, short period waves are

[13] Instrument factor relating vertical deflection of the instrument to sea water pressure is determined from laboratory calibration.

more effective in slowing up, or halting operations, than are those of longer periods. In the foregoing Cuttyhunk observations, the average period of bottom waves was about two times that at the surface, and for Bermuda about three times. The application of the modified hyperbolic cosine law to that part of the mechanical periodogram analysis below the computed average bottom period will, in some cases, serve to indicate the approximate average surface period. However it is a matter of individual judgment rather than one for which general rules can be laid down.

PART II

RESULTS OF SEA SURFACE ROUGHNESS DETERMINATIONS IN THE VICINITY OF WOODS HOLE, MASSACHUSETTS AND BERMUDA

CONTENTS

I. INTRODUCTION

The results presented here comprise data on the state of the sea surface in the vicinity of Woods Hole (off Cuttyhunk Island, Massachusetts) July 1946 to May 1947, and off Bermuda, British West Indies, February 1947 to May 1947, obtained from analyses of automatic wave recordings. At both locations measuring elements of the wave recording instruments were located on the sea bottom (approximate depth 75 feet at Cuttyhunk and 120 feet at Bermuda) and electrically connected to shore recorders several miles away[1]. The instrument operational schedules were continuous twenty minute recordings every two hours at Bermuda and every six hours at Cuttyhunk.

A total of 639 records from Cuttyhunk and 1022 from Bermuda were analyzed. The instrument records were scaled for pressure wave height (H_h) and wave period (T_h), at depth of instrumentation, h. The wave heights at the sea surface (H_s) are then computed by

$$H_s = 1.35\, H_h \operatorname{Cosh}\left(\frac{2\pi}{\lambda} h\right)$$

where

$$\lambda = \frac{g}{2\pi} T^2$$

In this equation, the factor 1.35 has been experimentally determined at both instrument sites, by comparison of simultaneous near bottom and surface wave heights[2]. Its probable error is 0.037.

II. THE SEA SURFACE OFF CUTTYHUNK ISLAND JUNE 1946 TO MAY 1947

1. Daily Wave Heights and Wave Periods

The average eight hourly values of surface wave heights, the heights of highest one third surface waves and the bottom pressure wave periods, from June 20, 1946 to May 13, 1947, are illustrated by Figures 1 to 5. The data are broken down to conform with available instrument records as follows:

June 20 to July 30, 1946	Figure 1
July 31 to Sept. 18, 1946	Figure 2
Sept. 19 to Nov. 8, 1946	Figure 3
Nov. 9 to Dec. 22, 1946	Figure 4
Mar. 17 to May 14, 1947	Figure 5

[1] The underwater pressure measuring instrument was designed and constructed at the Woods Hole Oceanographic Institution, under U. S. Navy contract NObs-2083.

[2] For discussion of this factor, see Part I.

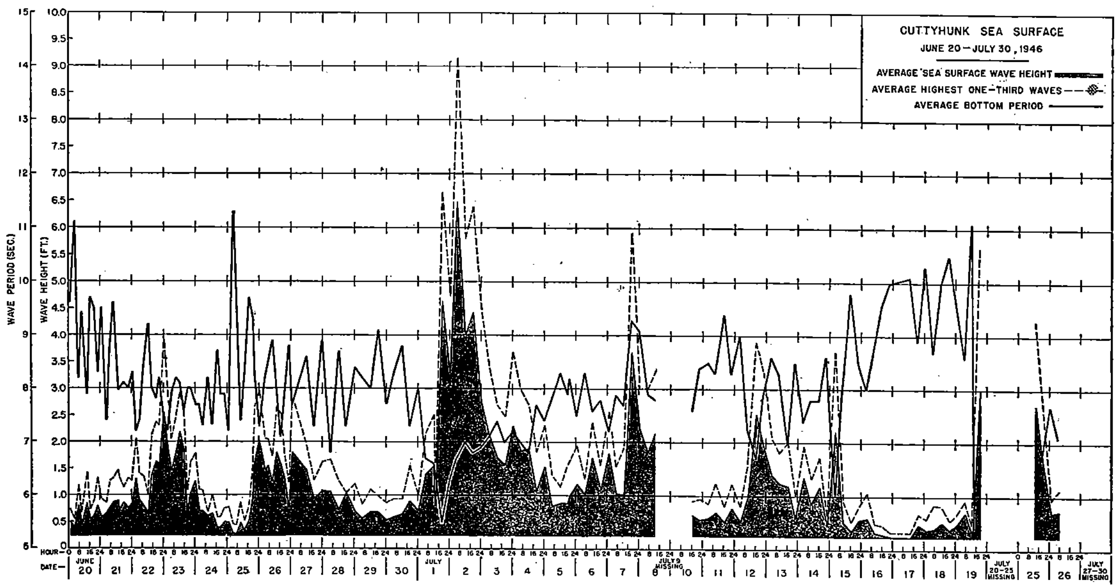

FIGURE 1

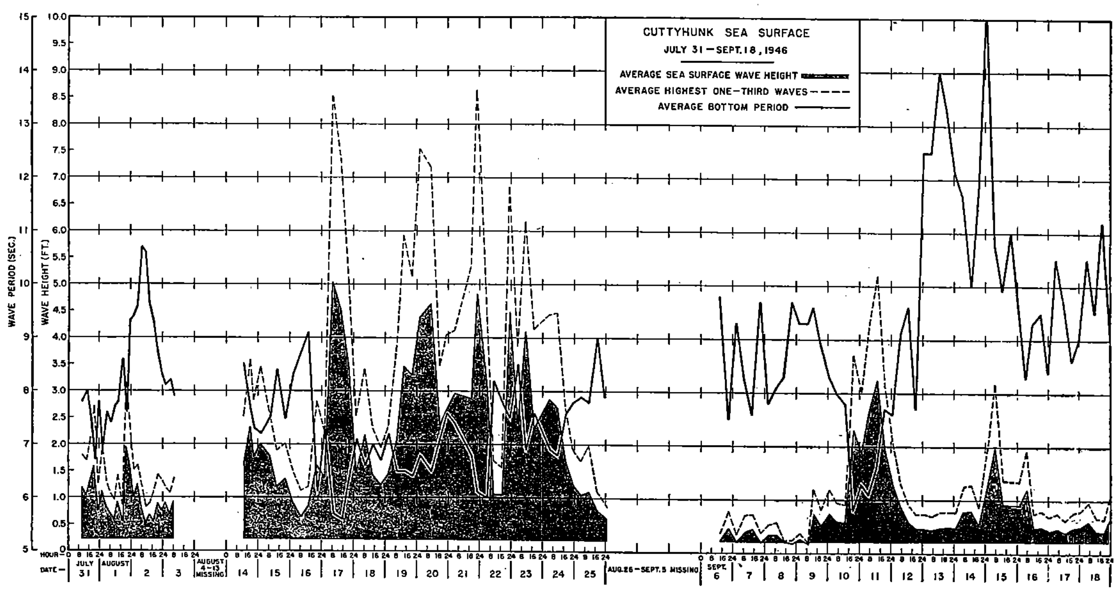

FIGURE 2

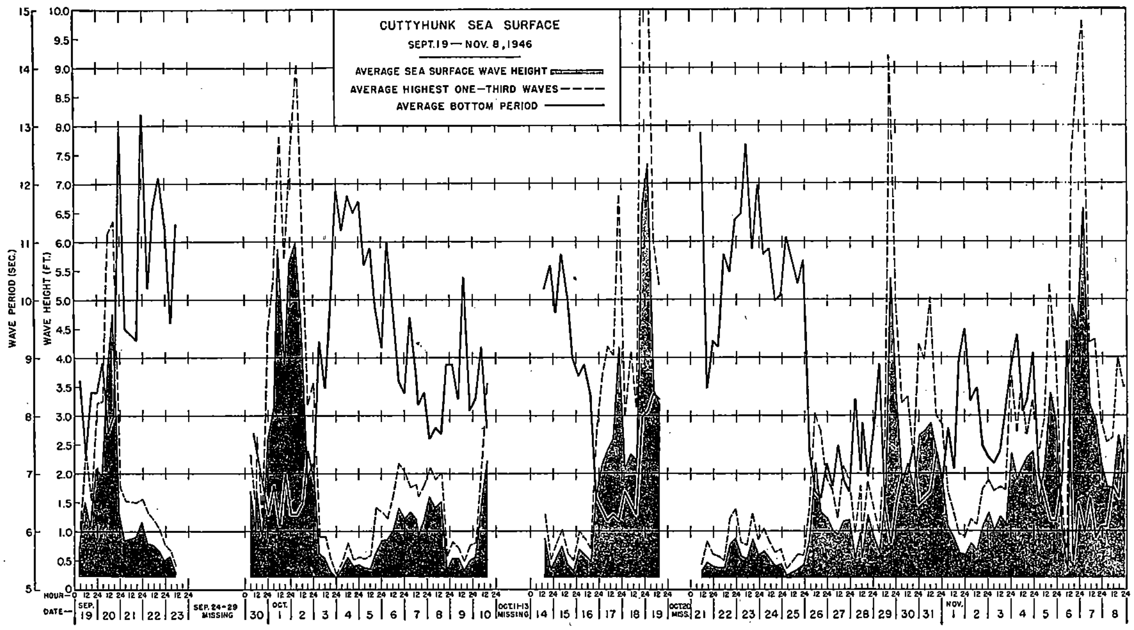

FIGURE 3

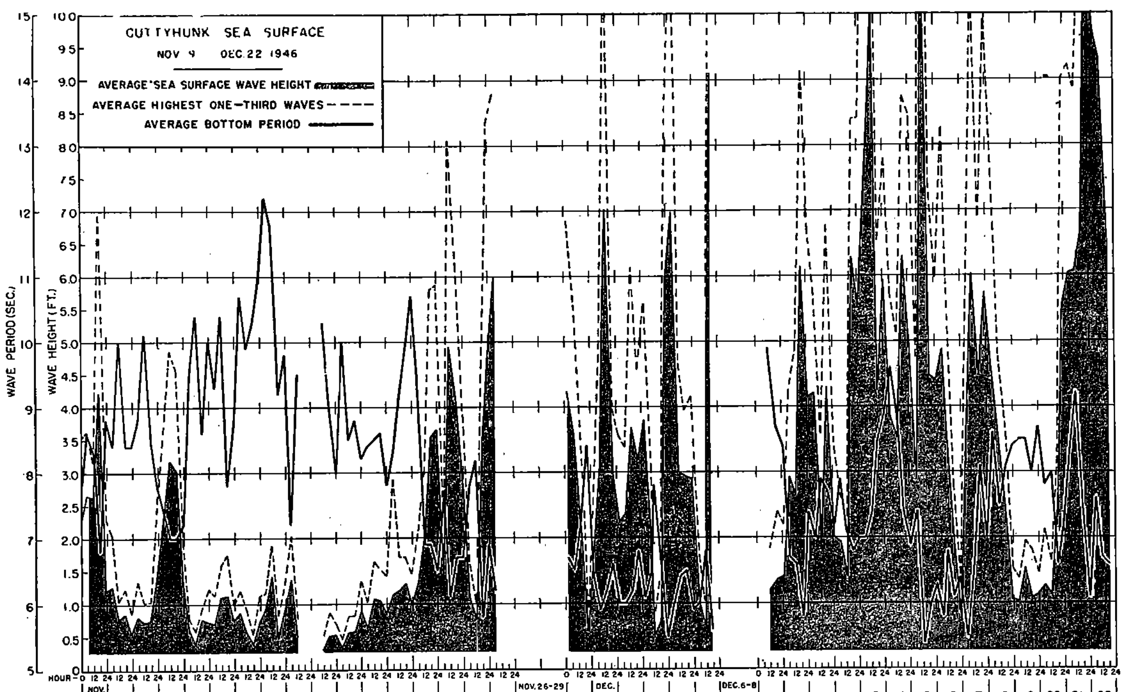

FIGURE 4

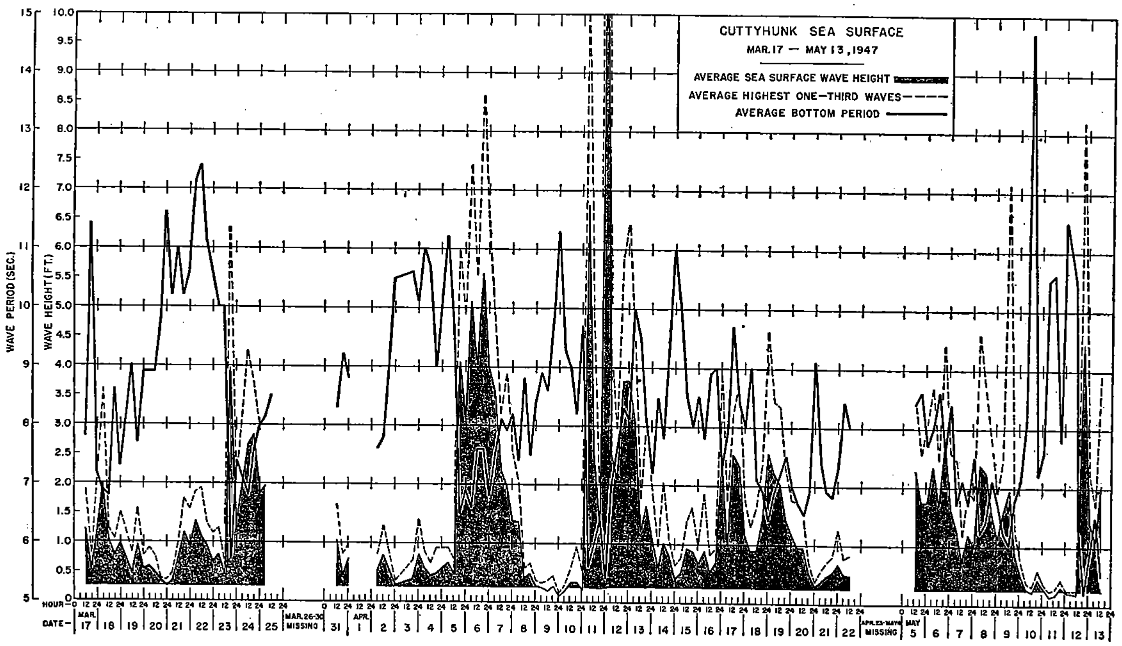

FIGURE 5

Interruptions in the data occurred when the wave recorder was non operational. In justification to the instrument, it need be pointed out that data for this analyses were obtained during a time of instrument experimentation, and not during a controlled observational program. Had the latter been possible, interruptions would have been far less frequent.

2. Monthly Sea Surface Wave Heights

Figures 6 to 14 illustrate the percentage frequency, and the cumulative percentage distribution of average sea surface wave heights, July 1946 to May 1947, inclusive, off Cuttyhunk Island. Because of their usefulness to future investigations, as well as for present practical problems, the charts are reproduced in detail. Each chart represents one month's data and its prognostic value need be, in absence of more complete information, judged accordingly.

From the cumulative curves, the percentage occurrences of average wave heights, above or below selected values can be read directly for each month. The percentage occurrence for a specified wave height is obtained from the frequency distribution curves. These data possess a usefulness for many practical Marine operations. In particular they permit estimates of operational conditions for surface boat work and for aircraft landings at sea, as well as for installation of marine structures.

The data on sea surface wave heights, summed in Table 1, provide a statistical picture of the monthly state of the sea surface in the vicinity of Woods Hole. In this table, the mode is the average wave height occurring most frequently, and is significantly less than the mean of all wave heights. The percentage occurrence of surface wave heights below the modal value, has been read directly from cumulative curves of Figures

6 to 14. Standard deviations and probable errors have been computed by formal methods. The coefficients of variability represent ratios of the standard deviations to the means. As is expected, the roughest seas, and the most rapidly changing conditions, occur during winter.

TABLE I

Month	Mean	Monthly Change	Probable Error	Standard Deviation	Mode	Monthly Change	Cumulative Occurrence Below Mode	Variability	Mean / Mode	Number of Observations
July	1.4154		0.0905	1.1849	0.6709		38 Percent	0.8371	2.1097	78
August	1.8563	1.311	0.1036	1.2285	0.9918	1.478	41 Percent	0.6618	1.8716	64
September	0.9222	0.497	0.0659	0.8299	0.5624	0.567	65 Percent	0.8999	1.6398	72
October	1.5538	1.649	0.0986	1.4908	0.7026	1.249	45 Percent	0.9595	2.2115	104
November	1.6845	1.084	0.0942	1.3749	0.9277	1.320	53 Percent	0.8162	1.8158	97
December	3.7506	2.227	0.1984	2.6476	1.0869	1.172	18 Percent	0.7059	3.4507	81
March	1.1200	0.299	0.0270	0.8142	0.7009	0.645	45 Percent	0.7270	1.5979	30
April	1.4150	1.263	0.1011	1.3407	0.6343	0.905	46 Percent	0.9475	2.2308	80
May	1.2667	0.895	0.1119	0.9530	0.4100	0.646	32 Percent	0.7523	3.0895	33

Surface wave height monthly summary. Cuttyhunk: July 1946 to May 1947.

3. MONTHLY SEA BOTTOM WAVE PERIODS

One of the characteristics of surface waves is that the radii, and consequently velocities, of the individual water particles (orbital velocities) decrease rapidly with depth. Thus pressure variations resulting from the vertical accelerations become imperceptible at, and below, depths of minimum motion. Theoretically it can be shown that at depths of one-half the wave length ($\lambda/2$) pressure variations from surface waves are negligible. Hence, off Cuttyhunk Island, with the pressure measuring instrument at a depth of 75 feet, waves of lengths less than 150 feet, or periods, $T = \sqrt{\frac{300\pi}{g}} = 5.2$ seconds, are not recorded at instrument depth. It is to be expected that periods deduced from bottom pressure recordings will represent upper limits of sea surface conditions, and, in general, the average period of the sea surface will be less than that given by bottom pressure records.

Regardless of this discrepancy, records of bottom wave pressures are useful, if for no other reason in that they are the only permanent continuous records obtainable some distance from shore. They take on an added significance when longer period waves dominate the sea surface, such as originate during storms over the high seas. Figures 15 to 23 illustrate the percentage frequency and the cumulative frequency distributions of average recorded bottom pressure wave periods for July 1946 to May 1947, in the Cuttyhunk vicinity. Each chart represents one month's data, and like those for average sea surface wave heights, the percentages of given wave periods above or below a specified value can be read directly. The prognostic value of these curves, limited to one month's information, must be judged accordingly. The data are useful for practical operational purposes, as well as for comparative studies with other areas.

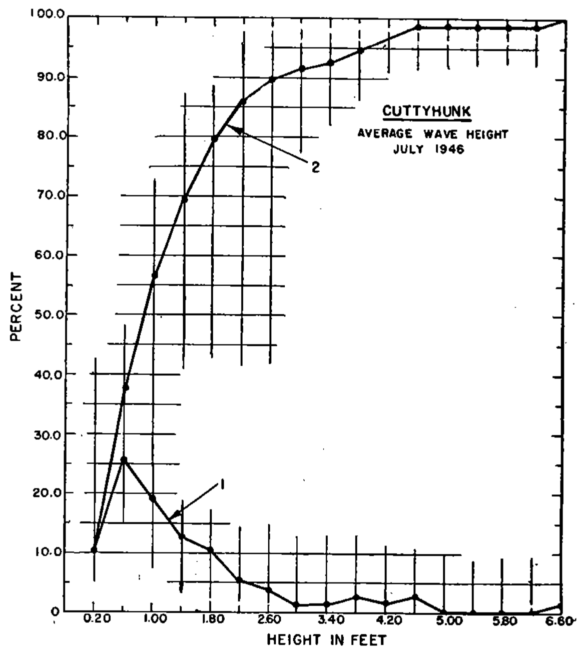

FIG. 6. Average Surface Wave Heights, Cuttyhunk, July 1946. 1 = frequency distribution; 2 = cumulative distribution.

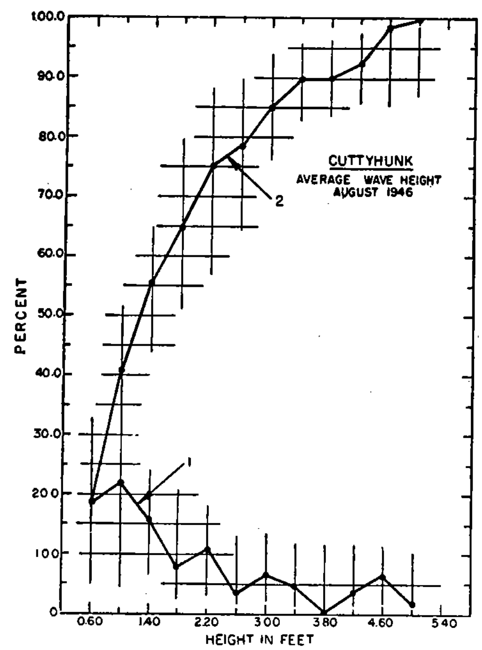

FIG. 7. Average Surface Wave Heights, Cuttyhunk, August 1946. 1 = frequency distribution; 2 = cumulative distribution.

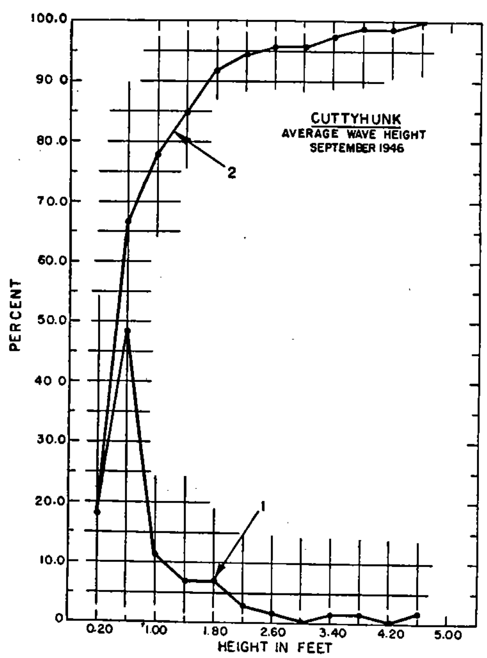

FIG. 8. Average Surface Wave Heights, Cuttyhunk, September 1946. 1 = frequency distribution; 2 = cumulative distribution.

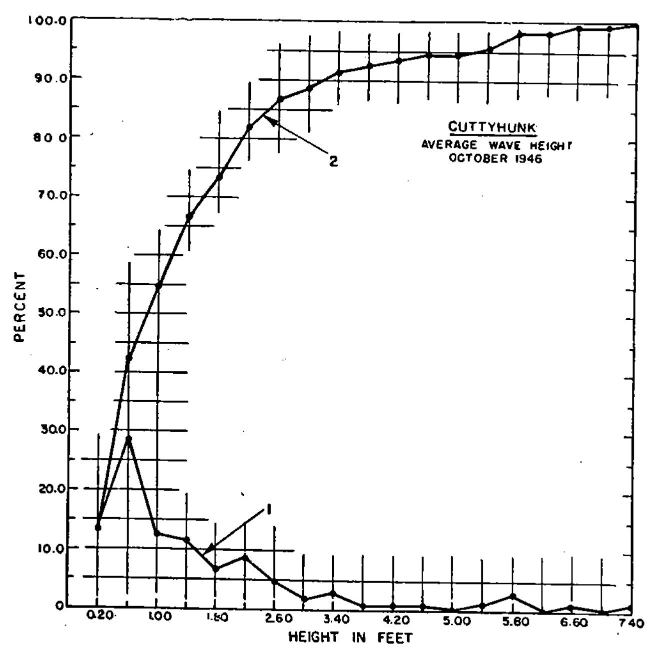

FIG. 9. Average Surface Wave Heights, Cuttyhunk, October 1946. 1 = frequency distribution; 2 = cumulative distribution.

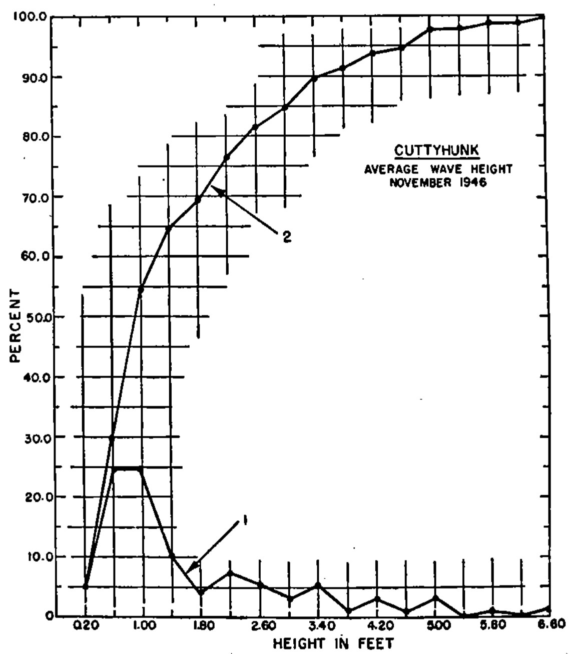

FIG. 10. Average Surface Wave Heights, Cuttyhunk, November 1946. 1 = frequency distribution; 2 = cumulative distribution.

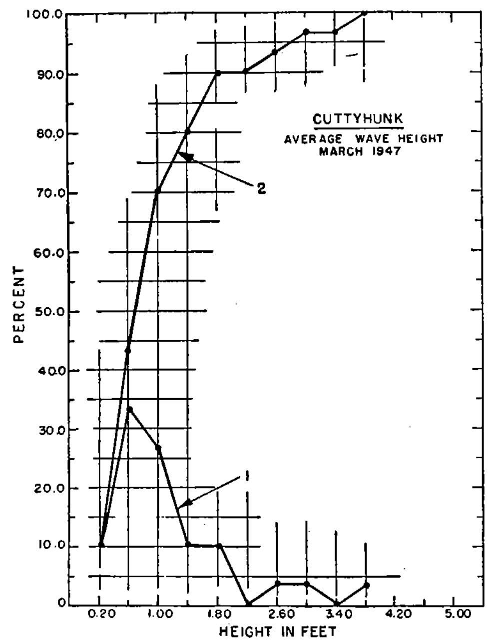

FIG. 12. Average Surface Wave Heights, Cuttyhunk, March 1947. 1 = frequency distribution; 2 = cumulative distribution.

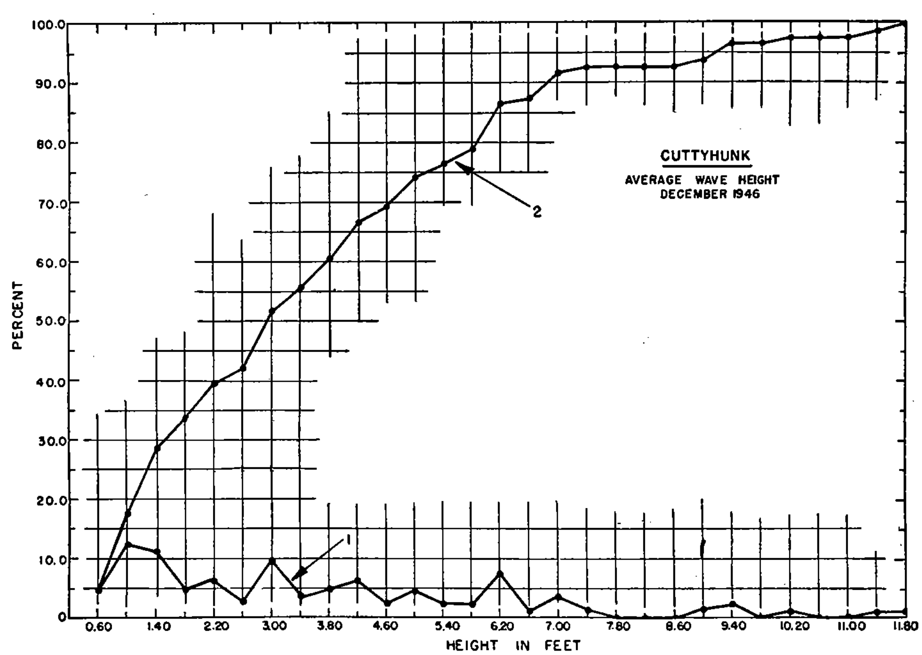

FIG. 11. Average Surface Wave Heights, Cuttyhunk, December 1946. 1 = frequency distribution; 2 = cumulative distribution.

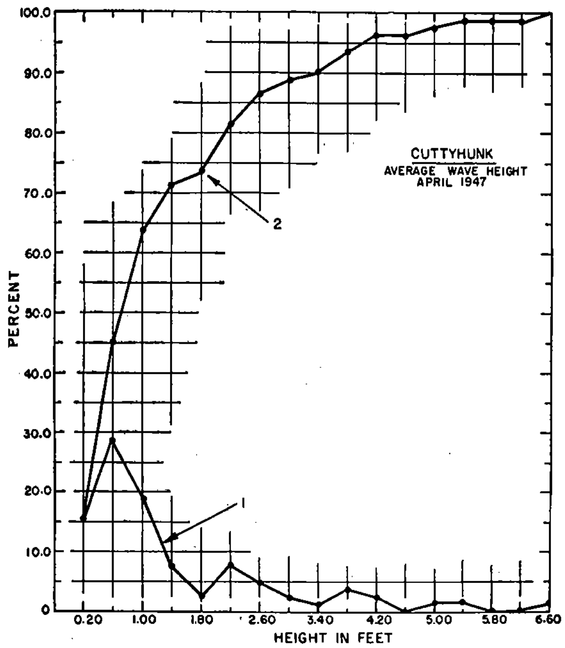

FIG. 13. Average Surface Wave Heights, Cuttyhunk, April 1947. 1 = frequency distribution; 2 = cumulative distribution.

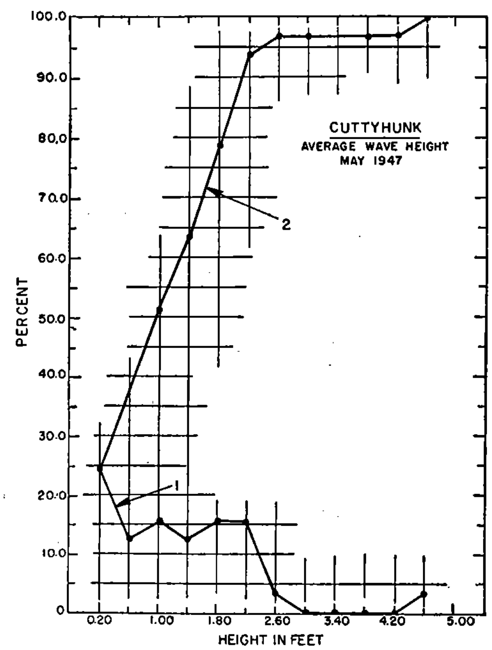

FIG. 14. Average Surface Wave Heights, Cuttyhunk, May 1947. 1 = frequency distribution; 2 = cumulative distribution.

The data of bottom wave periods are summed in Table 2. Like that for average sea surface wave heights, it possesses statistical significance. The most frequently occurring wave period (mode) is usually lower than the average, and higher mean periods seem to characterize Autumnal and Spring conditions, coinciding with off shore storm development.

TABLE 2

MONTH	MEAN	MODE	STANDARD DEVIATION	PROBABLE ERROR
July	7.994	7.729	1.097	0.084
August	7.727	7.700	1.091	0.092
September	9.507	9.600	1.944	0.155
October	8.620	6.737	1.828	0.121
November	8.219	6.813	1.421	0.097
December	7.460	6.729	1.283	0.096
March	9.650	8.833	1.795	0.221
April	8.294	8.200	1.368	0.103
May	7.841	6.722	1.952	0.229

Bottom wave period monthly summary; Cuttyhunk: July 1946 to May 1947.

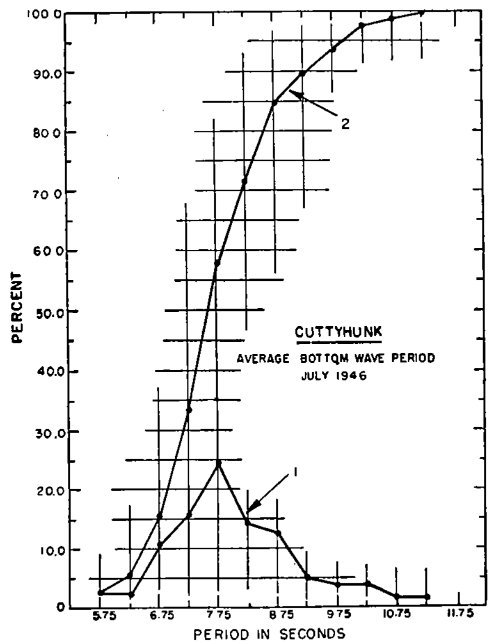

FIG. 15. Average Bottom Wave Periods, Cuttyhunk, July 1946. 1 = frequency distribution; 2 = cumulative distribution.

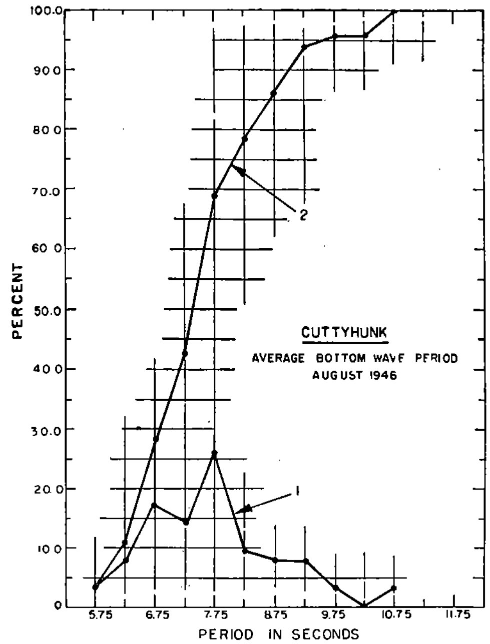

FIG. 16. Average Bottom Wave Periods, Cuttyhunk, August 1946. 1 = frequency distribution; 2 = cumulative distribution.

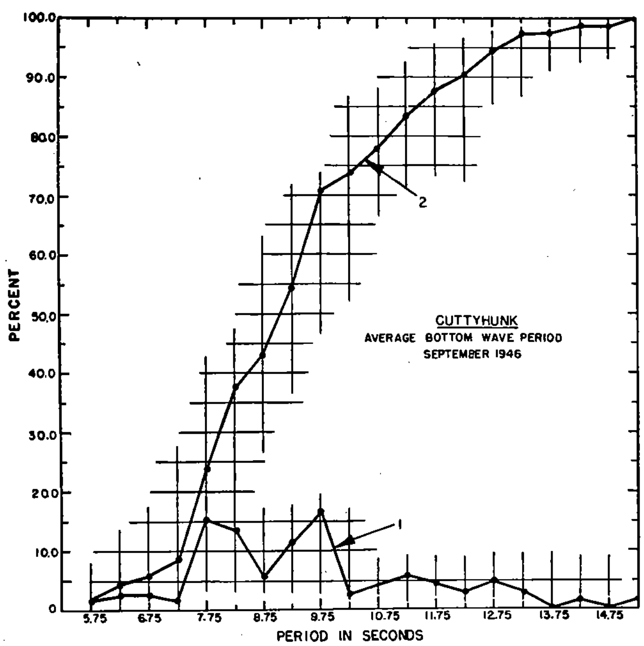

FIG. 17. Average Bottom Wave Periods, Cuttyhunk, September 1946. 1 = frequency distribution; 2 = cumulative distribution.

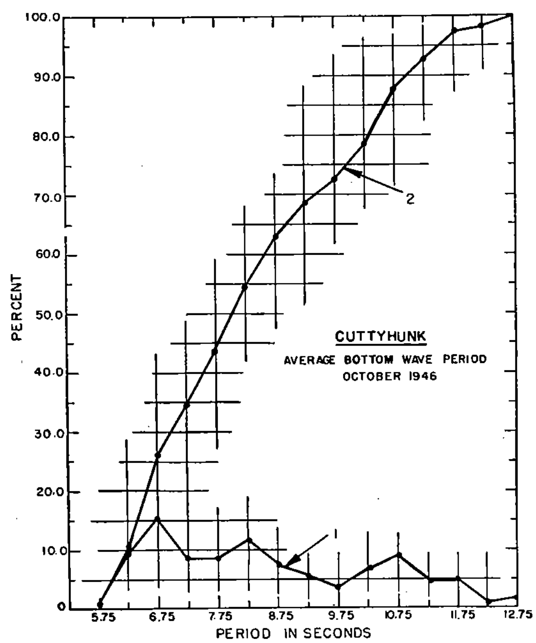

FIG. 18. Average Bottom Wave Periods, Cuttyhunk, October 1946. 1 = frequency distribution; 2 = cumulative distribution.

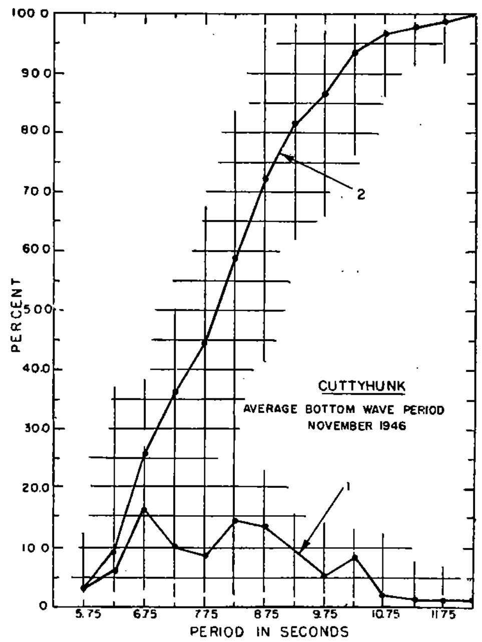

FIG. 19. Average Bottom Wave Periods, Cuttyhunk, November 1946. 1 = frequency distribution; 2 = cumulative distribution.

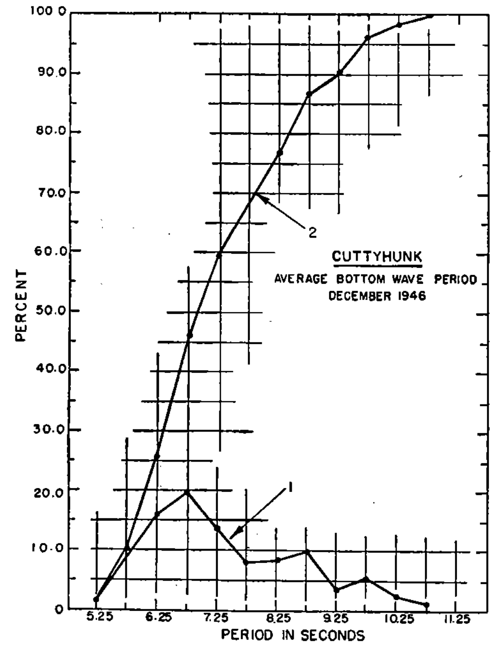

FIG. 20. Average Bottom Wave Periods, Cutty hunk, December 1946. 1 = frequency distribution; 2 = cumulative distribution.

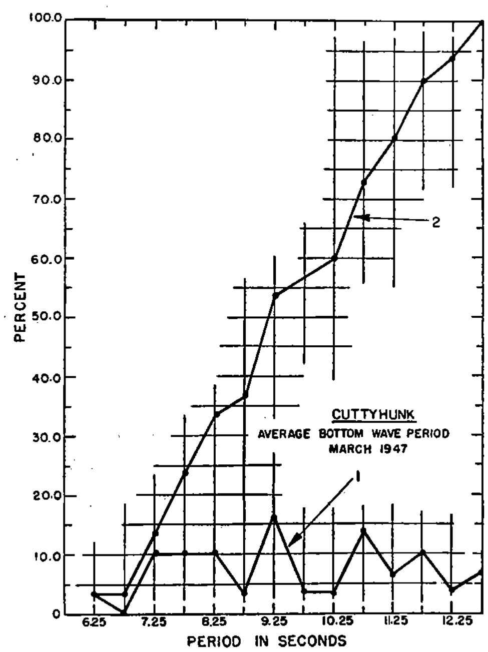

FIG. 21. Average Bottom Wave Periods, Cuttyhunk, March 1947. 1 = frequency distribution; 2 = cumulative distribution.

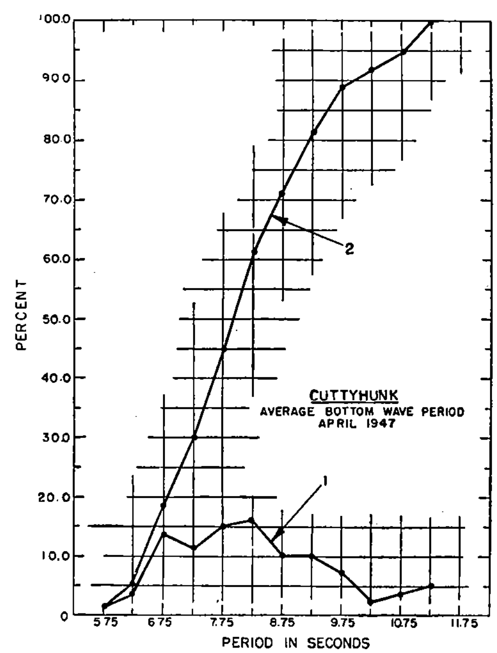

FIG. 22. Average Bottom Wave Periods, Cuttyhunk, April 1947. 1 = frequency distribution; 2 = cumulative distribution.

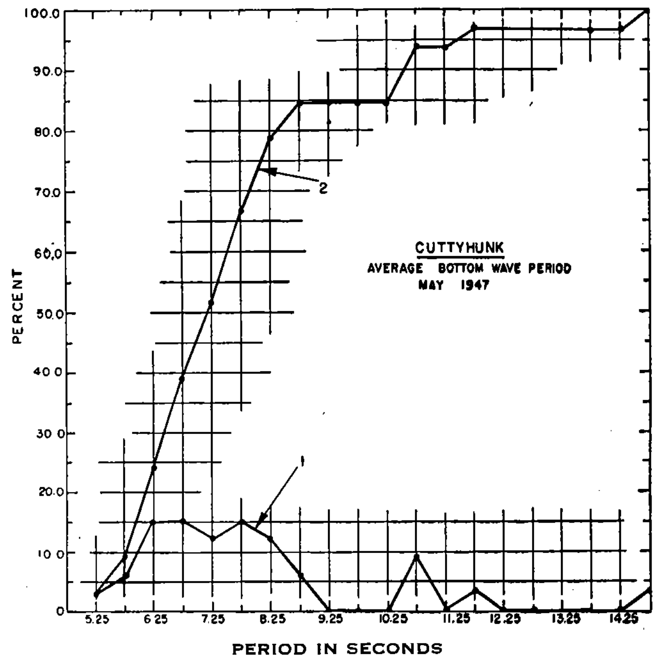

FIG. 23. Average Bottom Wave Periods, Cuttyhunk, May 1947. 1 = frequency distribution; 2 = cumulative distribution.

III. THE SEA SURFACE OFF BERMUDA, FEBRUARY TO MAY 1947

1. Daily Wave Heights and Wave Periods

Average six-hourly values of surface wave heights, the heights of the highest one third surface waves, and the bottom pressure wave periods, for the interval 12 February to 31 May 1947, are illustrated by Figures 24 to 27. The data are broken down to conform with instrument recordings as follows:

February 12 to March 11, 1947	Figure 24
March 13 to April 12, 1947	Figure 25
April 14 to May 13, 1947	Figure 26
May 13 to May 31, 1947	Figure 27

The values are based on means of two-hourly recordings. Interruptions in the data are due to the instrument being non operational. As in the case of Cuttyhunk, the data were obtained during experimental development of the instrument.

2. MONTHLY SEA SURFACE WAVE HEIGHTS

Figures 28 to 31 illustrate the percentage frequency and the cumulative percentage distribution of average sea surface wave heights from February to May, 1947. From these curves the average wave heights above or below selected values, and percentage occurrences can be obtained. The data are useful to practical marine operations, as well as for sea surface studies. Because of short duration, their prognostic value is restricted.

The data on sea surface wave heights, summarized in Table 3, provide a statistical picture of the monthly state of the sea surface from February to May in the Bermuda area, the statistics are identical with those of Table 1.

TABLE 3

MONTH	MEAN	MONTHLY CHANGE	PROBABLE ERROR	STANDARD DEVIATION	MODE	MONTHLY CHANGE	CUMULATIVE OCCURRENCE BELOW MODE	VARIABILITY	MEAN / MODE	NUMBER OF OBSERVATIONS
February	1.860		0.094	1.599	0.796		36 Percent	.8597	2.3367	133
March	1.429	.7683	0.040	1.078	0.770	.9673	48 Percent	.7544	1.8558	335
April	0.801	.5605	0.017	0.446	0.697	.9052	60 Percent	.5568	1.1492	305
May	0.825	1.0300	0.017	0.400	0.706	1.0129	64 Percent	.4848	1.1686	249

Surface wave height monthly summary. Bermuda: February 1947 to May 1947.

During the brief observation time, mean surface wave heights decreased from 1.86 in February to 0.80 in April, after which a levelling off occurs, to be probably followed by smaller declines in summer. Coincident with the decline in roughness, and a diminishing variability, is a relatively constant modal value. The data show the most frequently occurring mean surface wave height changed from only 0.8 feet in winter to 0.7 feet in spring. The occurrence of seas below modal height, however, increased from 36 percent in February to 64 percent in May. The transition from winter to spring values is reflected by increased occurrence of modal wave heights, rather than by changes in the mode itself. This suggests the mode as an important operational factor to be forecasted.

The frequency occurrence of any series of wave heights can be obtained from the cumulative wave height curves of Figures 28 to 31. From an operational standpoint the establishment of average 25, 50 and 75 percent values of wave heights is desirable.

3. MONTHLY SEA BOTTOM WAVE PERIODS

Due to the damping of pressure variations below depths of minimum orbital velocities, surface waves with lengths less than 240 feet, or periods less than 6.35 seconds, are not expected to be recorded at instrument depth off Bermuda (120 feet).

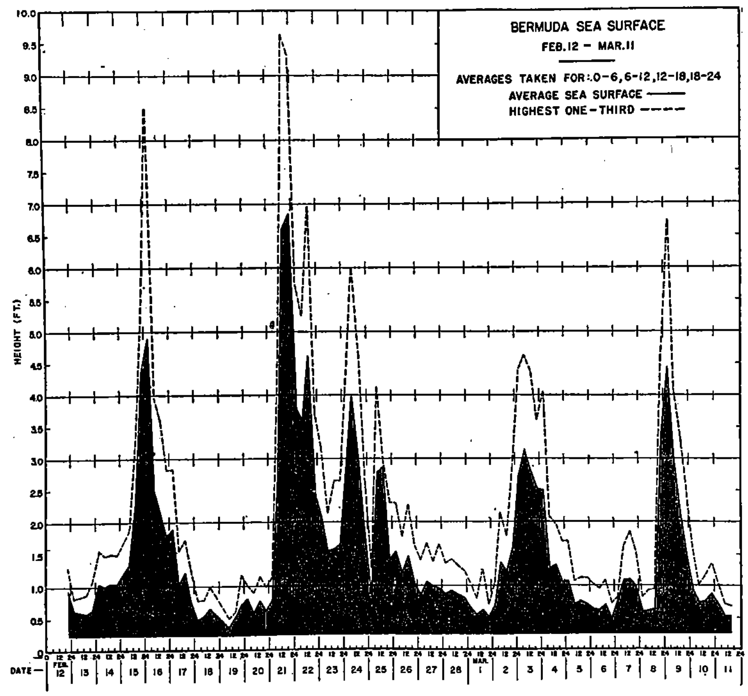

FIGURE 24

Figures 32 to 35 illustrate the percentage frequency and the cumulative frequency distributions of recorded bottom pressure periods. The arrangements of the charts is similar to those for Cuttyhunk data. The monthly bottom wave data, summarized in Table 4, are directly comparable to that for Cuttyhunk in Table 2. The mode generally falls below the mean; from March to May the mode remained at 9.3 seconds whereas the mean increased from 9.6 to 10.3 seconds. Although the significance of this is not known, it indicates the mode as an important operational factor for forecasting.

TABLE 4

MONTH	MEAN	MODE	STANDARD DEVIATION	PROBABLE ERROR
February...........	9.626	9.727	0.922	0.054
March..............	9.577	9.245	1.282	0.047
April...............	9.827	9.305	1.346	0.052
May................	10.326	9.313	1.433	0.061

Bottom wave period monthly Summary; Bermuda: February 1947 to May 1947.

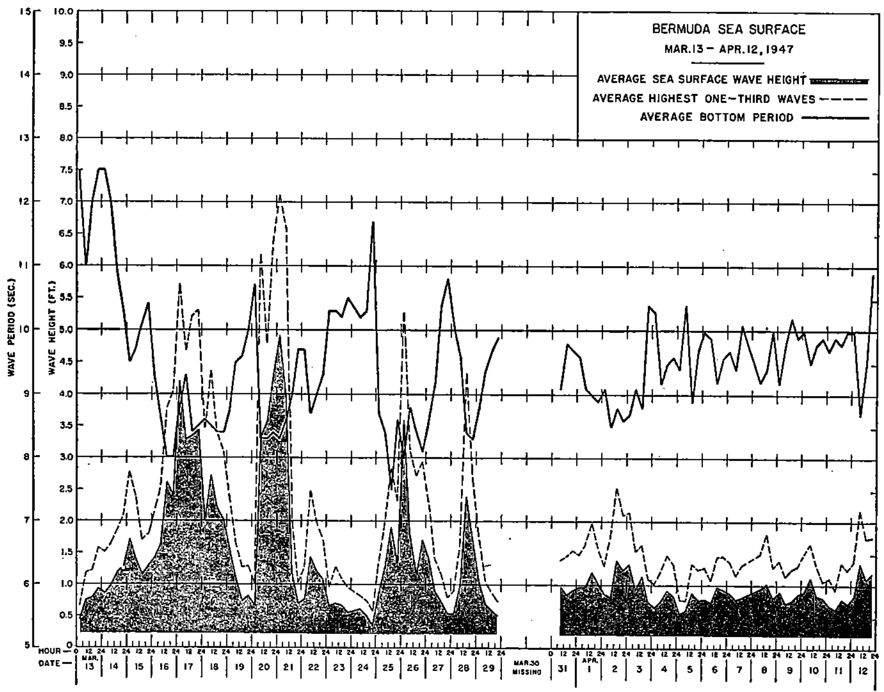

FIGURE 25

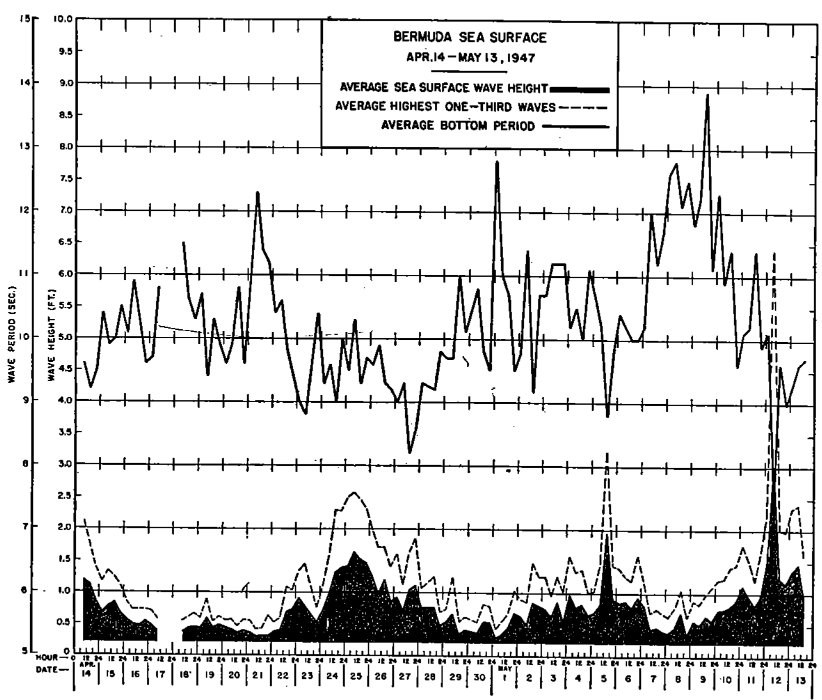

FIGURE 26

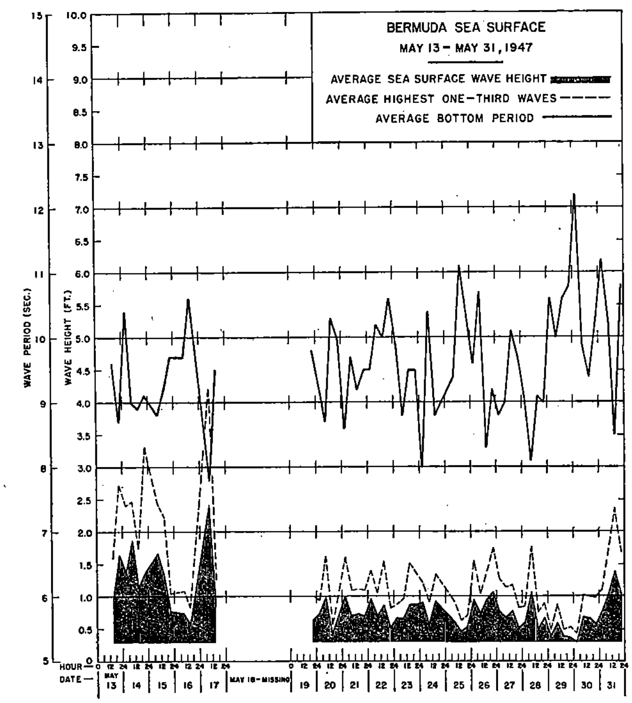

FIGURE 27

IV. INTERRELATIONSHIPS OF WAVE CHARACTERISTICS

1. Wave Height and Wave Period

From the foregoing, it is apparent that at both localities, the higher wave heights are usually accompanied by lower periods, and vice versa (Figures 1 to 5 and 24 to 27, Table 1 to 4). For example, at Cuttyhunk, where mean surface wave heights increased continuously from 0.92 feet in September to 3.75 feet in December, the accompanying mean bottom wave periods diminished from 9.51 to 7.46 seconds; and at Bermuda, where the mean surface wave height diminished from 1.86 feet in February to 0.83 feet in May, the accompanying mean wave period increased from 9.63 to 10.33 seconds.

This is the observed overall pattern; with the exception of occasional departures, higher surface wave heights are accompanied by lower periods. Having established this, the problem of rate of change of wave height and wave period in relation to the state of the sea is next considered.

2. The Growth and Decay of Sea Surface Wave Heights and Wave Periods

An ideal solution to the practical problem of sea surface roughness would be one establishing statistical relationships between the rate of change of sea surface wave

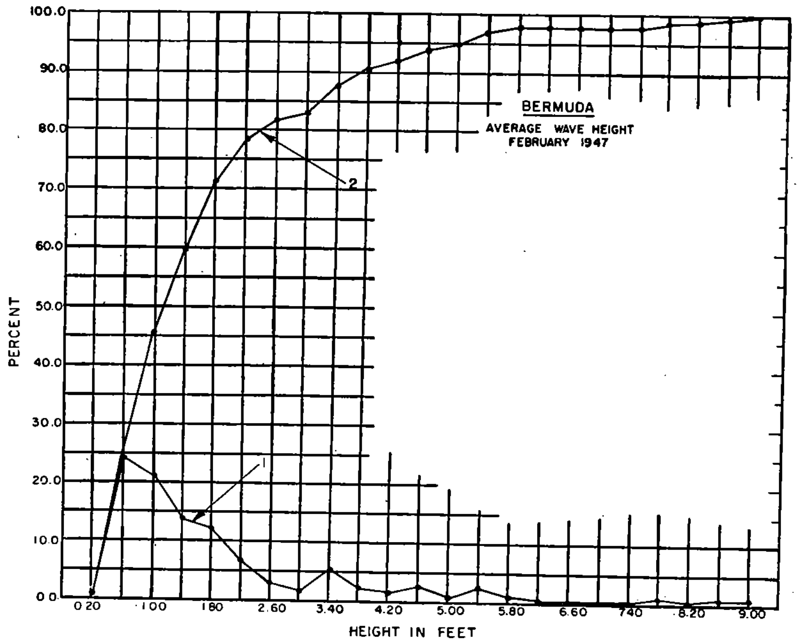

FIG. 28. Average Surface Wave Heights, Bermuda, February 1947. 1 = frequency distribution; 2 = cumulative distribution.

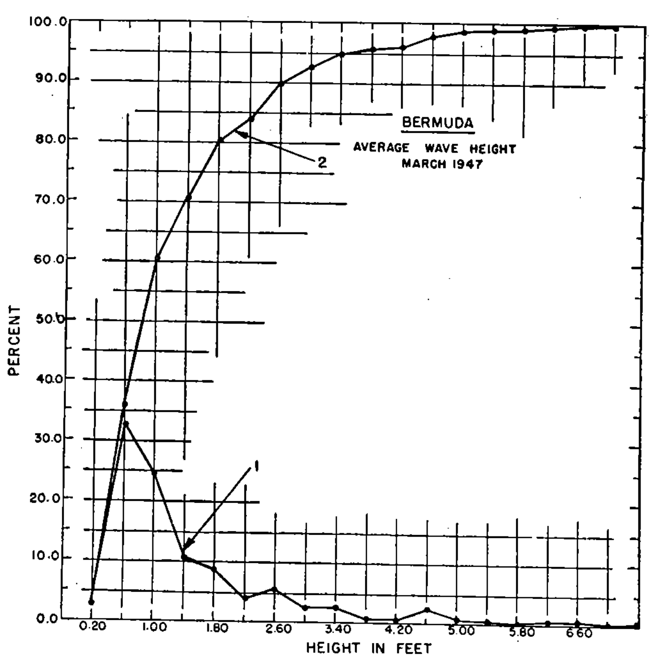

FIG. 29. Average Surface Wave Heights, Bermuda, March 1947. 1 = frequency distribution; 2 = cumulative distribution.

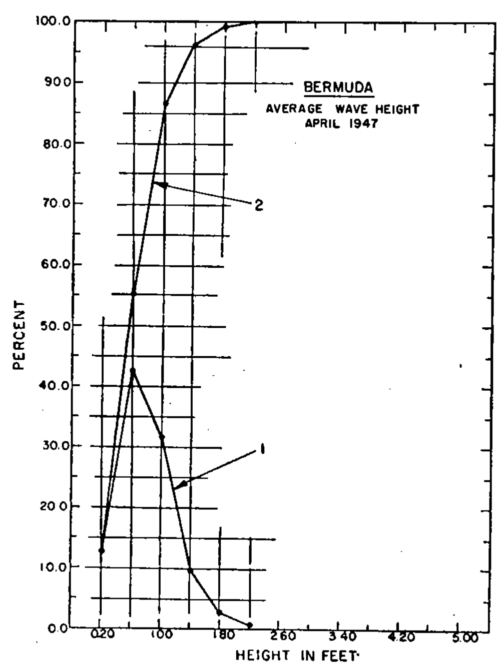

FIG. 30. Average Surface Wave Heights, Bermuda, April 1947. 1 = frequency distribution; 2 = cumulative distribution.

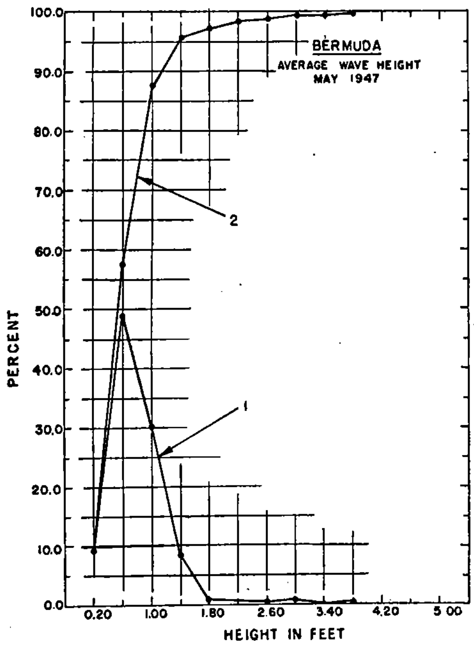

FIG. 31. Average Surface Wave Heights, Bermuda, May 1947. 1 = frequency distribution; 2 = cumulative distribution.

height and wave period and the state of the sea. The data available at present do not permit an extensive analysis as they are limited to the few months, including a single series of observations for each month. Consequently, with the exception of generalized relationships brought out, computed monthly values are valid only insofar as the single series of observations conform to near normal values for the months in question. The following approach has been to first, evaluate the patterns of rates of change of wave heights and wave periods, by describing their frequency distributions, and second, to examine the results in relation to the state of the sea.

Rates of change of Wave height $\left(\frac{\Delta H}{\Delta t}\right)$ and Wave period $\left(\frac{\Delta T}{\Delta t}\right)$ were computed from the six-hourly records at Cuttyhunk and the two-hourly records at Bermuda. In the treatment of the general cases, frequency distributions for separate, positive, and negative values of each region were formed. The statistical quantities describing them are tabulated in Table 5.

TABLE 5

	CUTTYHUNK				BERMUDA			
	$+\frac{\Delta H}{\Delta t}$	$-\frac{\Delta H}{\Delta t}$	$+\frac{\Delta T}{\Delta t}$	$-\frac{\Delta T}{\Delta t}$	$+\frac{\Delta H}{\Delta t}$	$-\frac{\Delta H}{\Delta t}$	$+\frac{\Delta T}{\Delta t}$	$-\frac{\Delta T}{\Delta t}$
Mean	0.159	0.136	0.166	0.182	0.178	0.200	0.479	0.475
Standard deviation	0.221	0.171	0.168	0.171	0.234	0.242	0.425	0.479
Probable error	0.009	0.006	0.006	0.007	0.007	0.008	1.014	0.015
Mode	0.05	0.05	0.05	0.05	0.05	0.05	0.10	0.10
Cumulative occurrence to mode	30%	30%	20%	20%	25%	22%	16%	18%
50 Percent cumulative occurrence	0 to 0.08	0 to 0.08	0 to 0.13	0 to 0.14	0 to 0.10	0 to 0.12	0 to 0.36	0 to 0.32
75 Percent cumulative occurrence	0 to 0.185	0 to 0.170	0 to 0.22	0 to 0.25	0 to 0.20	0 to 0.25	0 to 0.65	0 to 0.65
Maximum value	1.5	1.5	1.5	1.5	1.9	1.9	1.9	1.9

Summary of statistical properties of rates of change of wave height $\left(\text{increasing} = +\frac{\Delta H}{\Delta t}\right.$ foot/hour, decreasing $\left.= -\frac{\Delta H}{\Delta t}\right)$ and rate of change of wave period $\left(\text{increasing} = +\frac{\Delta T}{\Delta t}\right.$ sec/hour, decreasing $\left.= -\frac{\Delta T}{\Delta t}\right)$ for Cuttyhunk (July 1946 to May 1947) and Bermuda (February 1947 to May 1947).

The situation demonstrated by the statistical properties of Table 5 indicates a remarkable uniformity in rates of Growth and Decay of sea surface roughness at both Bermuda and Cuttyhunk. At both localities, rates of Growth $\left(+\frac{\Delta H}{\Delta t}\right)$ and Decay $\left(-\frac{\Delta H}{\Delta t}\right)$ are about equal, with an over all range from 0 to 1.5 feet per hour at Cuttyhunk and from 0 to 1.9 feet per hour for Bermuda. The most frequently occurring changes in sea surface wave heights are very small, both localities indicate modal values of about 0.05 feet per hour occurring 22 to 30 percent of the time. Rates of changes of sea surface wave heights occurring 50 percent of the time do not appear to exceed 0.12 feet per hour, and those for 75 percent of the time, not exceeding 0.25 feet per hour. At both Cuttyhunk and Bermuda, the sea surface wave height increases and diminishes at approximately the same average rate $\left(\text{mean } \frac{\Delta H}{\Delta t} \doteq 0.168 \text{ ft/hr.}\right)$. At both localities rates of change of sea surface wave height exceeding one foot per hour, appear to occur less than two percent of the time.

Time changes in wave period are also small; the maximum recorded for Cuttyhunk was 1.5 seconds per hour, and for Bermuda 1.9 seconds per hour. The modal values of 0.05 to 0.10 seconds per hour occurred 16 to 20 percent of the time. At Cuttyhunk wave periods changed less than 0.14 seconds per hour and at Bermuda less than 0.36 seconds per hour for 50 percent of the time. Wave period changes exceeding one second per hour, occurred less than one percent of the time at Cuttyhunk and 10 percent of the time at Bermuda.

It is apparent that only in exceptional cases does the state of the sea surface change rapidly. Furthermore, the almost continuously changing wave heights and wave periods are related. This is brought out by Figures 36 and 37, where the following relationships have been derived for the two localities by least square fits of straight lines. Thus, for Cuttyhunk:

$$\frac{\Delta H}{\Delta t} = -0.421 \frac{\Delta T}{\Delta t}$$

for Bermuda:

$$\frac{\Delta H}{\Delta t} = -0.255 \frac{\Delta T}{\Delta t}$$

Differences in the coefficients may, in part at least, arise from different surface-bottom wave period relationships, discussed in Part I. However, the above, together with statistical characteristics, tabulated in Table 5, have short interval forecasting significance, in that they provide a means for estimating, within restricted limits, the state of the sea to be expected a few hours hence. A somewhat more complete picture is given in the following section where rates of growth and decay of sea surface wave heights are related to the existing state of the sea.

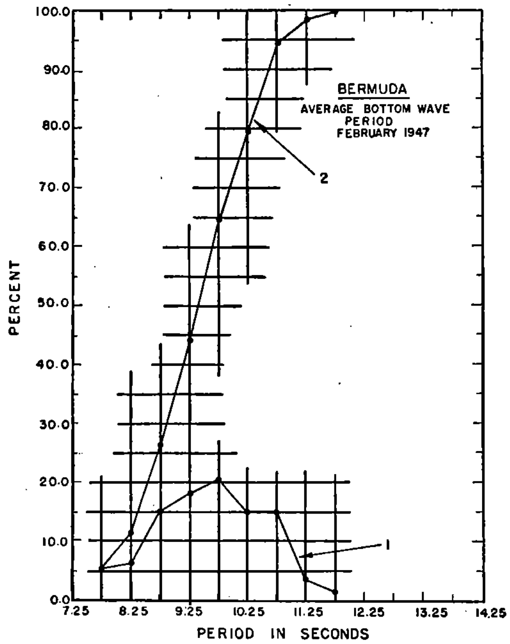

FIG. 32. Average Bottom Wave Periods, Bermuda, February 1947. 1 = frequency distribution; 2 = cumulative distribution.

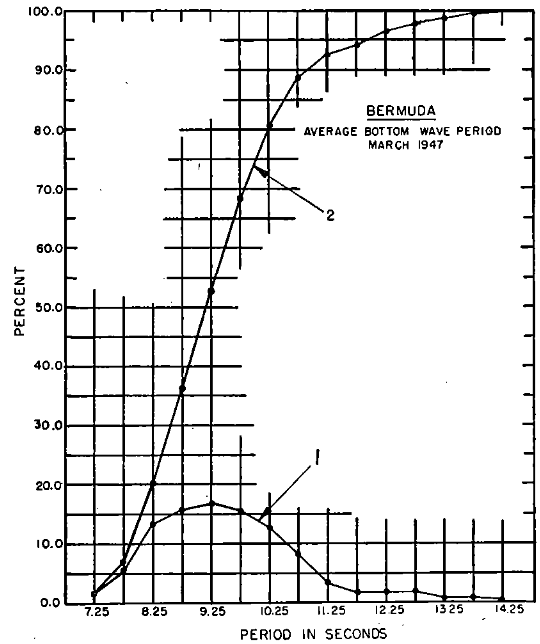

FIG. 33. Average Bottom Wave Periods, Bermuda, March 1947. 1 = frequency distribution; 2 = cumulative distribution.

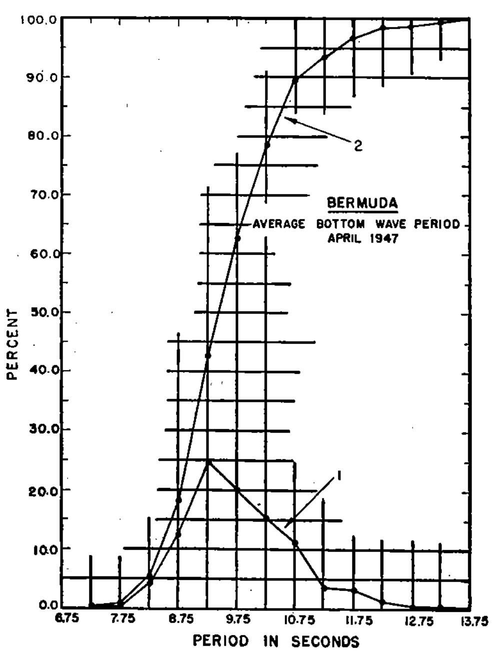

FIG. 34. Average Bottom Wave Periods, Bermuda, April 1947. 1 = frequency distribution; 2 = cumulative distribution.

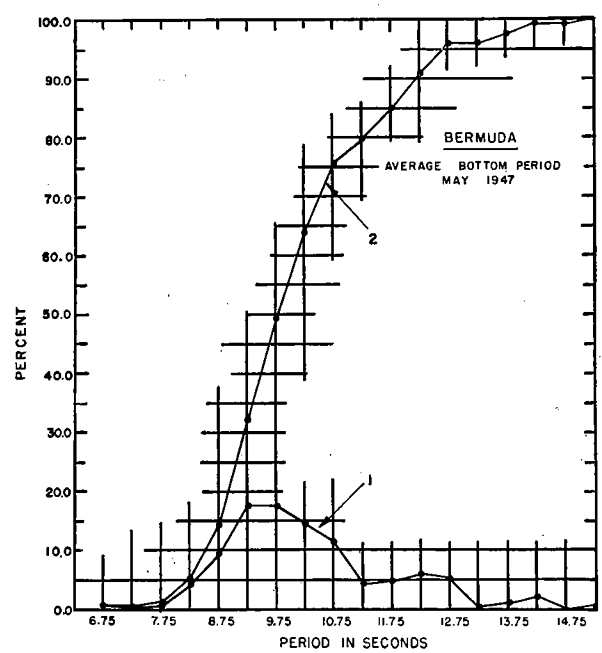

FIG. 35. Average Bottom Wave Periods, Bermuda, May 1947. 1 = frequency distribution; 2 = cumulative distribution.

3. The Growth and Decay of Sea Surface Wave Heights in Relation to the Mean Wave Height

The percentage occurrence of wave height increases and decreases, relative to existing surface wave heights, are summarized in Table 6. The figures show the percentage of increases or decreases, and those within the modal limits of plus or minus 0.05 foot per hour. Computations are for the entire observation period, and hence include effects of seasonal variations. They provide a generalized picture of expected changes in sea surface wave heights relative to the mean wave heights.

Pertinent points, illustrated by Table 6, are that rates of change of sea surface wave heights, within the modal value, occur chiefly when wave heights are less than three feet. At greater wave heights the tendency is for sea surface roughness to change at greater rates, particularly as regards its decreasing tendency for the highest mean wave heights. The increase in sea surface roughness at Bermuda is somewhat more restricted than at Cuttyhunk; wave heights in excess of one foot show a diminishing roughness tendency which exceeds the increasing tendency whereas at Cuttyhunk this change occurs after two foot levels have been reached. The situation appears to arise from differences in the local meteorological situations.

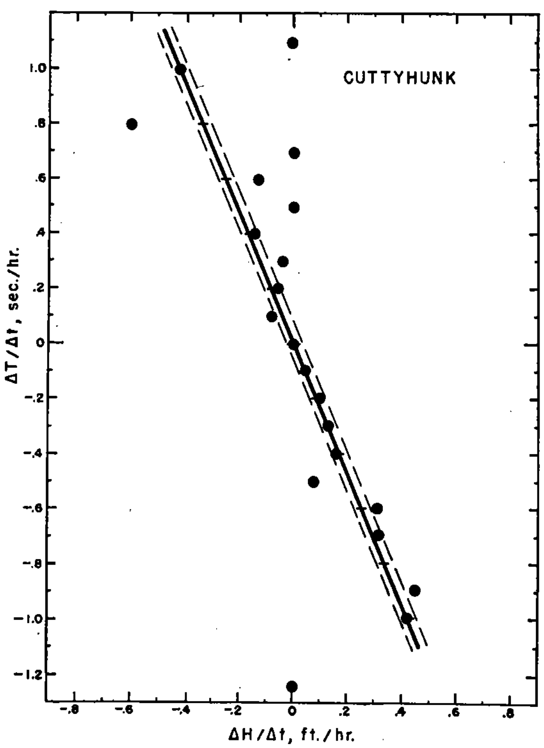

FIG. 36. Relationship between rates of change of Wave Height $\left(\frac{\Delta H}{\Delta t}\right)$ and Wave Period $\left(\frac{\Delta T}{\Delta t}\right)$ Cuttyhunk (See text).

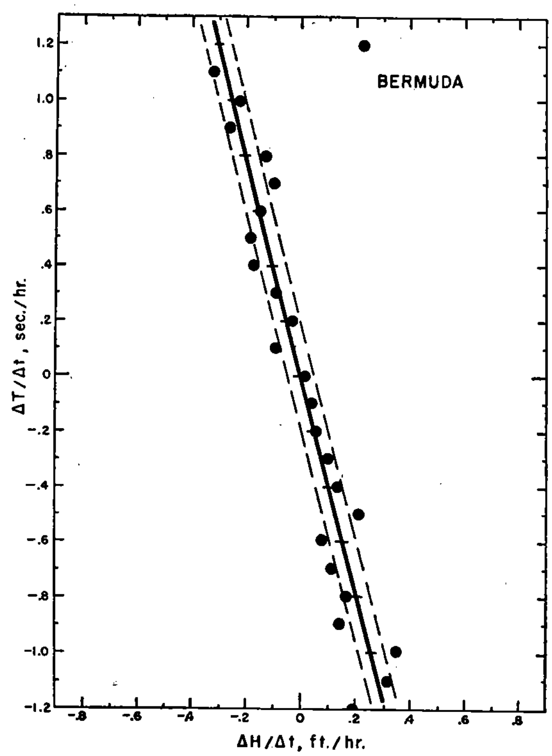

FIG. 37. Relationship between rates of change of Wave Height $\left(\frac{\Delta H}{\Delta t}\right)$ and Wave Period $\left(\frac{\Delta T}{\Delta t}\right)$ Bermuda (See text).

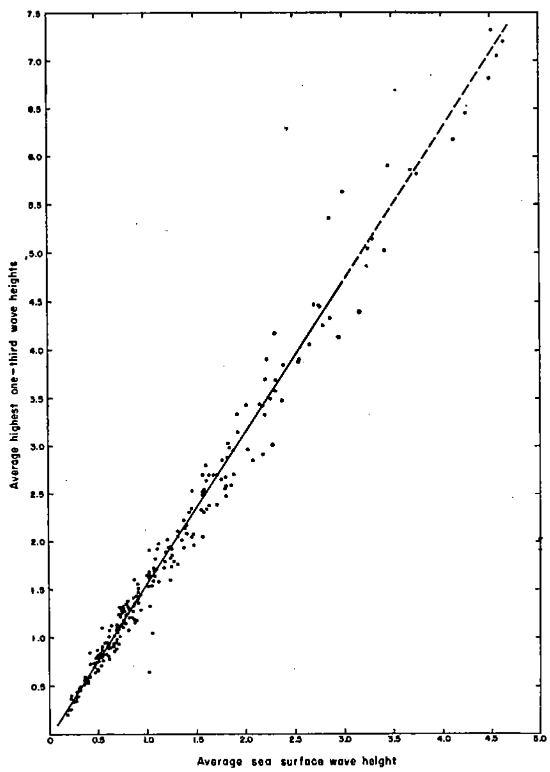

FIG. 38. Relationship between average wave heights and average heights of highest one third waves, Cuttyhunk sea surface (See text).

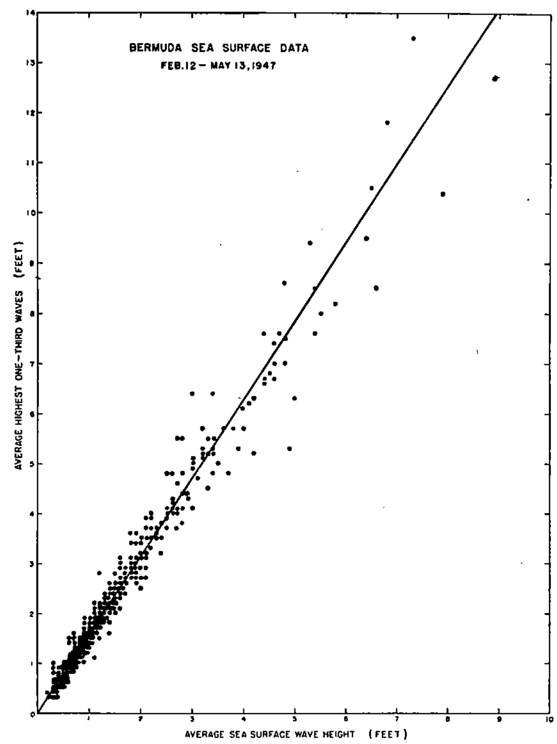

FIG. 39. Relationship between average wave heights and average heights of highest one third waves, Bermuda sea surface (See text).

The forecasting value of this data is that it enables a crude estimate to be made of the future probable changes in sea surface wave heights. It is comparable to that for climatological data, and need be used with caution, since individual values may be subject to wide variations. It is, however, better than the guess work which usually characterizes Marine operations.

4. Relation of Mean Wave Heights to the Mean of the Highest One Third Waves at the Sea Surface; the Operational Wave Height

The situation discussed so far is based on the mean wave heights for two minute intervals, from two and six-hourly records, as the case may be. Coincident with the determinations of mean wave heights, the means of the highest one third surface waves were also calculated. These latter values are termed OPERATIONAL WAVE HEIGHTS in that they better represent practical effects of sea surface roughness than do mean values of all wave heights. The deduction is arbitrary, but from the standpoint of practical operations it provides a somewhat better index of sea surface roughness. Hence, in the following, analyses of operational significance are given in terms of the OPERATIONAL WAVE HEIGHTS.

TABLE 6

CUTTYHUNK

MEAN SURFACE WAVE HEIGHT (FEET)	$-\frac{\Delta H}{\Delta t} > 0.05$ FEET/HOUR	$\frac{\Delta H}{\Delta t} = -0.05$ to $+0.05$	$+\frac{\Delta H}{\Delta t} > 0.05$ FEET/HOUR	NUMBER OF OBSERVATIONS
> 7	100	0	0	6
6–7	69.3	7.7	23.1	13
5–6	63.7	0	36.4	11
4–5	68.7	9.4	22.0	32
3–4	51.4	17.1	31.6	35
2–3	50.0	30.5	19.5	82
1–2	38.3	32.6	29.4	181
0–1	8.0	65.2	26.7	276

BERMUDA

> 6	100	0	0	7
5–6	100	0	0	5
4–5	81.1	0	18.6	16
3–4	54.6	9.1	36.2	22
2–3	59.4	6.8	34.0	59
1–2	53.8	12.6	33.7	253
0–1	25.1	34.6	40.4	547

Summary of rates of change of sea surface wave height as related to the average surface wave height. Percentage summary according to sea surface wave heights decreasing more than 0.05 feet per hour $\left(-\frac{\Delta H}{\Delta t} > 0.05\right)$, sea surface wave height changing between limits of -0.05 feet per hour to $+0.05$ feet per hour $\left(\frac{\Delta H}{\Delta t} = -0.05 \text{ to } +0.05\right)$, or sea surface wave height increasing more than 0.05 feet per hour $\left(+\frac{\Delta H}{\Delta t} > 0.05 \text{ feet per hour}\right)$. Figures showing percentage occurrence of each orbitary division of rate of change, total 100 percent for each surface wave height division.

The data from both areas show a significant relation between the averages of all wave heights and the averages of the highest one third wave heights. This is illustrated for Cuttyhunk (639 observations) and Bermuda (1022 observations) by Figures 38 and 39. Independent fits of straight lines by least squares to both sets of data give identical results. Thus:

Mean of Highest One Third Waves = 1.57 x Mean Wave Height

Hence, it is reasonable to compute operational wave heights by multiplying mean wave heights by the factor of 1.57. As for other factors derived in this study, they are considered pertinent only to the localities for which derived. It is considered too early to generalize on the meager results of these and similar investigations.

V. THE SUMMER STATE OF THE SEA SURFACE IN THE VICINITY OF WOODS HOLE (CUTTYHUNK, ISLAND)

Because of the practical value to summer marine activities in this vicinity, a special summary of sea surface roughness, observed during the summer of 1946, is given in Table 7, and Figures 40 and 41. Although the degree of sea surface roughness is relative to the specific type of operation and type of surface craft, the basic data of Table 7 enable the quantitative formulation of operational requirements and permit limited comparisons of operating conditions in different regions, hence eliminating guess work. In Table 7, surface wave heights and bottom wave periods are statistically described, and are representative of the summer state of the sea in the Woods Hole vicinity insofar as the one series of observations is representative.

TABLE 7

	MEAN SURFACE WAVE HEIGHT	MEAN BOTTOM WAVE PERIOD	OPERATIONAL SURFACE WAVE HEIGHT
Mean	1.25 feet	8.12 secs.	1.96
Probable Error	0.043	0.045	
Standard Deviation	0.947	0.995	
Mode	0.67	7.71	1.05
Cumulative occurrence below mode	44%	58%	
Variability	0.757	0.123	
Mean/Mode	1.86	1.06	
25 percent cumulative occurrence	0 to 0.40 feet	5.5 to 7.0	0 to 0.63
50 percent cumulative occurrence	0 to 0.80 feet	5.5 to 7.5	0 to 1.26
75 percent cumulative occurrence	0 to 1.40 feet	5.5 to 8.3	0 to 2.20
Number of Observations	220	220	220

Summary Summer sea surface conditions in Woods Hole vicinity (off Cuttyhunk Island) 20 June to 9 September 1946.

The cumulative curves of Figures 40 and 41 permit estimates of mean summer wave height conditions, the latter are transformed to operational wave heights by a factor of 1.57. Figures 40 and 41 in combination with Table 7, represent the present state of our knowledge of basic sea surface roughness conditions in this locality, and arepresented in such a fashion as to enable estimates of specific operational conditions.

A scheme for representing practical operational conditions, by combinations of wave height and wave period is illustrated by Table 8. The breakdown into favorable, unfavorable and conditional situations is with reference to small boat work; it is arbitrary and presented chiefly as an example of a practical method. The boundaries of wave height — wave period combinations may be selected with reference to any operation on the basis of experience.

In the Cuttyhunk wave height — wave period diagram of Table 8, the average of the highest one third waves (operational wave heights) are balanced against associated bottom wave periods. The diagram includes data for the summer season only. Each figure represents the percent occurrence of the height-period combinations. Summing

TABLE 8

WAVE HEIGHT	WAVE PERIODS 5-6	6-7	7-8	8-9	9-10	10-11	11-12	PERCENT SUMMARY
10-11		0.4						0.4
9-10								
8-9	0.4							0.4
7-8	0.9	1.8	0.4					3.1
6-7		0.8						0.8
5-6	0.5	1.8			0.4			2.7
4-5	0.4	2.3	1.7					4.4
3-4	0.4	1.4	4.9	0.9	0.5			8.1
2-3		3.5	10.7	3.6	0.4			18.2
1-2		1.8	18.7	12.5	3.6	0.4		37.0
0-1			4.4	8.4	8.9	1.9	1.3	24.9
PERCENT SUMMARY	2.6	13.8	40.8	25.4	13.8	2.2	1.3	100

Summary Sea Surface operating conditions in Percent of Time, Cuttyhunk, Summer 1946. Wave height (feet) = average highest one third wave during twenty minute intervals. Wave period (seconds) = average recorded at bottom. Small boat operating conditions: Favorable; bounded by solid lines; unfavorable; bounded by dashed lines.

values within the arbitrarily selected zones gives the following estimate of summer operating conditions.

Favorable Operating Conditions = 77 percent
Unfavorable Operating Conditions = 12 percent
Intermediate Operating Conditions = 11 percent

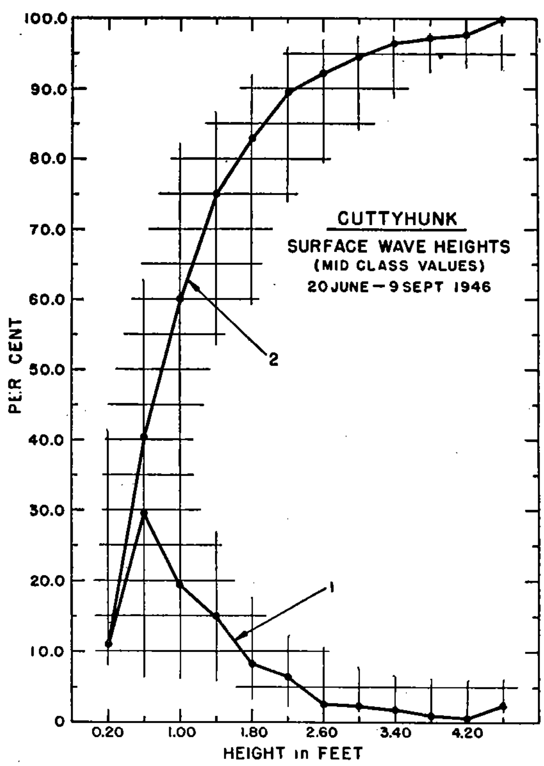

FIG. 40. Frequency distribution (1) and cumulative distribution (2) of surface wave heights, Cuttyhunk, summer 1946.

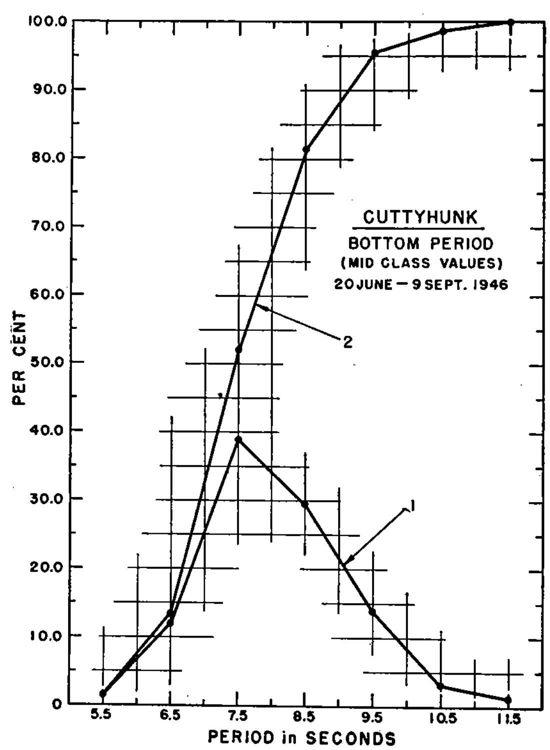

FIG. 41. Frequency distribution (1) and cumulative distribution (2) of bottom wave periods, Cuttyhunk, summer 1946.

Extension of this analysis scheme to all the Cuttyhunk and Bermuda data produces the seasonal summary of Table 9, permitting comparison of seasonal operating conditions. If data of this type were available throughout the world, it would be possible to set up a useful schedule for any type of marine operation. The results of Table 9, obtained by the method of Table 8, show that for Cuttyhunk the 75 percent favorable operating conditions of spring and summer are reduced to about 50 percent during the autumn.

TABLE 9

LOCATION	SEASON	SEA SURFACE CONDITION		
		Favorable	Unfavorable	Intermediate
Cuttyhunk	Spring	73	13	14
	Summer	77	12	11
	Autumn	55	31	14
Bermuda	Winter	74	11	15
	Spring	90	1	9

Seasonal Sea Surface operating conditions in Percent of Total Time at Cuttyhunk and Bermuda, computed from boundaries established for Table 6.

At Bermuda, the 75 percent favorable operating conditions of winter, increase to 90 percent in spring. Data from the two localities are not directly comparable because of differences in the bottom-surface period relationships, as brought out in Part I.

VI. THE SEA SURFACE PATTERN

Previous discussion has been chiefly concerned with sea surface roughness conditions from the operational standpoint. The purpose of this section is now to illustrate certain features of sea surface patterns significant to design of surface craft and Marine structures. It is based on the basic data of Tables 1, 2, 3, and 4, to which are added the data of extreme conditions. The occurrence of extreme conditions of sea surface roughness, (highest waves) is more significant to planning the design of surface craft than to operational planning. Data for Cuttyhunk and Bermuda are tabulated in Table 10. Formulae used in the computation are given in the legend; wave heights are operational heights (average heights of highest one third waves).

TABLE 10

CUTTYHUNK

Month	Operational Wave Height Mean	Operational Wave Height Mode	Wave Length Mean	Wave Length Mode	Wave Velocity Mean	Wave Velocity Mode	Mean Op. Wave Hgt / Mean Wave Length	Mode Op. Wave Hgt / Mode Wave Length
Cuttyhunk								
July.....	2.22	1.05	327.2	305.9	24.2	23.4	0.0068	0.0034
August...	2.91	1.56	305.7	303.6	23.4	23.3	0.0095	0.0051
September	1.45	0.88	462.8	471.9	28.8	29.1	0.0031	0.0018
October..	2.44	1.10	380.4	232.4	26.1	20.4	0.0064	0.0048
November	2.64	1.46	345.9	237.7	24.9	20.6	0.0077	0.0061
December	5.89	1.71	284.9	231.8	22.6	20.4	0.0207	0.0074
March...	1.76	1.10	476.8	399.5	29.2	26.8	0.0037	0.0028
April.....	2.22	1.00	352.2	344.3	25.1	24.8	0.0063	0.0029
May.....	1.99	0.64	314.8	231.4	23.8	20.4	0.0063	0.0028
Bermuda								
February.	2.92	1.25	474.4	484.4	29.2	29.5	0.0062	0.0026
March...	2.24	1.21	469.6	437.6	29.0	28.0	0.0048	0.0028
April.....	1.26	1.09	494.4	443.3	29.8	28.2	0.0025	0.0025
May.....	1.30	1.11	545.9	444.1	31.3	28.2	0.0024	0.0025

Summary of sea surface pattern for Cuttyhunk computed from basic data of tables 1, 2, 3, and 4. Operational Wave Height = 1.57 x mean wave height; wave length = 5.12 T^2, wave velocity = 3.03 T (T = period). Heights and lengths in feet, velocities in nautical miles per hour. Wave Height/Wave Length = Steepness.

1. The Cuttyhunk Sea Surface Pattern

Table 10 gives mean and modal wave length data, as derived from the Cuttyhunk basic data (Tables 1 and 3). Because surface waves of less than 150 feet in length were not recorded by the Cuttyhunk pressure instruments (depth 75 feet), the mean and modal values of wave length represent upper limits.

The range of mean wave lengths for this area is from 285 to 475 feet, compared with modal values of 232 to 472 feet. The shorter winter wave lengths and greater wave heights, result in their being three to four times steeper than in summer. The

TABLE 11

	Cuttyhunk Spring	Cuttyhunk Summer	Cuttyhunk Autumn	Bermuda Winter	Bermuda Spring
Mean	3.486	3.179	4.927	3.809	2.355
Probable error	0.156	0.108	0.173	0.092	0.036
Standard deviation	2.730	2.644	4.393	2.936	1.096
Mode	1.561	1.622	1.756	1.930	2.348
Cumulative occurrence below mode	26%	28%	12%	30%	53%
25 percent cumulative occurrence	0 to 1.5	0 to 1.5	0 to 1.8	0 to 1.8	0 to 1.5
50 percent cumulative occurrence	0 to 2.8	0 to 2.5	0 to 2.9	0 to 2.8	0 to 2.3
75 percent cumulative occurrence	0 to 4.9	0 to 3.9	0 to 7.3	0 to 4.9	0 to 3.0
95 percent cumulative occurrence	0 to 9.0	0 to 9.5	0 to 14.5	0 to 10.5	0 to 4.5
Highest wave	16	15	22	17	7

Summary of maximum wave height statistics, derived from maximum wave heights of twenty minute recordings at Cuttyhunk and Bermuda (all data in feet) maximum mean winter steepness was 2.1 percent compared with a mean summer value of about 0.6 percent. The modal steepness values ranged from 0.2 percent in summer to 0.7 percent in winter.

The highest waves are those which occurred in each 20 minute recorded interval, every six hours. Results for Cuttyhunk observations are tabulated according to season in Table 11. The highest wave of each season is based on a single observation and may not be correct.

Mean values of the highest waves range from 2.1 times the mean wave height in autumn, to 2.7 in spring (see Table 1). Table 11 illustrates that at Cuttyhunk, highest waves encountered during spring and summer were about equal, and increased considerably during autumn. Comparison of summer mean and maximum wave heights, is obtained from mean values of Table 7 and maximum values of Table 11. Thus, the 25, 50 and 75 percent cumulative occurrence values show that heights of the highest waves to be about three times the mean wave heights and about twice the operational wave heights. It is to be expected ratios for other seasons are approximately the same.

Although not too much faith is attached to the seasonal highest wave values, they enable approximations of possible wave height-length ratios occurring in the waters of this vicinity. Thus associating the maximum highest wave height of 22 feet with a six second period gives a height-length ratio of 0.1195 or a steepness value of about 12 percent.

2. The Bermuda Sea Surface Pattern

Surface wave lengths less than 240 feet were not registered by the Bermuda wave recorder (depth 120 feet); consequently mean and modal values of Table 10 represent upper limits and are not directly comparable to those for Cuttyhunk. The shorter wave lengths and higher waves, result in greater wave steepness in winter. Thus, Table 10 shows mean wave lengths increased from about 470 feet in winter to 546 feet in spring and operational wave heights diminished from 0.6 percent to 0.2 percent.

Statistics characterizing the highest waves at Bermuda are tabulated in Table 11. Means of the highest wave heights for winter and spring are 2.3 to 2.9 times mean values of all waves, or approximately the same order of magnitude as for Cuttyhunk. The cumulative occurrence at the 25, 50, 75 and 95 percent levels show considerable falling off in heights of the highest waves during the transition from winter to spring. The winter maximum wave height of 17 feet associated with a six second period, gives a height length ratio of 0.092 or a steepness of about 9 percent.